Dissertation Defense

Eulerian series, zeta functions and the arithmetic of partitions

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Abstract: In this talk we prove theorems at the intersection of the additive and multiplicative branches of number theory, bringing together ideas from partition theory, q-series, algebra, modular forms and analytic number theory. We present a natural multiplicative theory of integer partitions (which are usually considered in terms of addition), and explore new classes of partition-theoretic zeta functions and Dirichlet series — as well as "Eulerian" q-hypergeometric series — enjoying many interesting relations. We find a number of theorems of classical number theory and analysis arise as particular cases of extremely general combinatorial structure laws.

Among our applications, we prove explicit formulas for the coefficients of the q-bracket of Bloch-Okounkov, a partition-theoretic operator from statistical physics related to quasi-modular forms; we prove partition formulas for arithmetic densities of certain subsets of the integers, giving q-series formulas to evaluate the Riemann zeta function; we study q-hypergeometric series related to quantum modular forms and the "strange" function of Kontsevich; and we show how Ramanujan's odd-order mock theta functions (and, more generally, the universal mock theta function g_3 of Gordon-McIntosh) arise from the reciprocal of the Jacobi triple product via the q-bracket operator, connecting also to unimodal sequences in combinatorics and quantum modular-like phenomena.

Tuesday, April 3, 2018, 3:00 pm
PAIS 280

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