Dissertation Defense

Monstrous moonshine, elliptic curves and vertex algebras

Victor Manuel Aricheta
Emory University

Abstract: In mathematics, moonshine refers to the unexpected connection between group theory and number theory. Monstrous moonshine is the first and best understood example of moonshine. It links the monster sporadic group to the modular j-function, a fact that we now know is explained by the presence of a certain algebraic structure called the moonshine module. Our comprehension of monstrous moonshine remains incomplete however, and in this talk we discuss several aspects of it that remain mysterious.

First we investigate a theorem and an observation of Ogg in 1975 that foreshadowed monstrous moonshine. In particular we generalize his theorem on supersingular j-invariants to supersingular elliptic curves with level structure. Ogg observedwhich we now know is partly explained by monstrous moonshinethat the level one case yields a characterization of the primes dividing the order of the monster. Here we show that the corresponding analyses for higher levels give analogous characterizations of the primes dividing the orders of other sporadic simple groups (e.g. baby monster, Fischer's largest group). More generally we characterize, in terms of supersingular elliptic curves with level, the primes arising as orders of Fricke elements in centralizer subgroups of the monster. This situates Ogg's theorem and observation in a broader setting.

Second we build on the study of Duncan, Griffin and Ono concerning the moonshine module. They studied its homogeneous subspaces, and discovered in particular that the moonshine module exhibits a curious property: its homogeneous subspaces tend to a multiple of the regular representation of the monster. We prove that an analogous result holds for any vertex operator algebra satisfying certain hypotheses, for which the moonshine module is the first natural example.

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Advisor: John Duncan

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