Algebra Seminar

Integers represented by positive-definite quadratic forms and Petersson inner products

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Abstract: We give a survey of results about the problem of determining which integers are represented by a given quaternary quadratic form Q. A necessary condition for $Q(x_1, x_2, x_3, x_4)$ to represent n is for the equation $Q(x_1, x_2, x_3, x_4) = n$ to have a solution with $x_1, x_2, x_3, x_4 \in Z_p$ for all p. But even when n is sufficiently large, this is not sufficient for Q to represent n. The form Q is anisotropic at the prime p if for $x_1, x_2, x_3, x_4 \in Z_p$, $Q(x_1, x_2, x_3, x_4) = 0$ implies that $x_1 = x_2 = x_3 = x_4 = 0$. Suppose that A is the Gram matrix for Q and $D(Q) = \det(A)$. We show that if $n >> D(Q)^{6+\epsilon}$, n is locally represented by Q, but Q fails to represent n, then there is an anisotropic prime p so that $p^2|n$ and np^{2k} is not represented by Q for any $k \geq 1$. We give sharper results when $D(Q)$ is a fundamental discriminant and discuss applications to universality theorems like the 15 and 290 theorems of Bhargava and Hanke.

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