Abstract: The hyperelliptic curve given by the equation $y^2 = f(x)$ with coefficients in $\mathbb{Q}$ has an unusual arithmetic property: it admits infinitely many points with coordinates in quadratic extensions of $\mathbb{Q}$ (namely $(a, \sqrt{f(a)})$). Hindry, motivated by arithmetic questions about modular curves, asked if the only curves that possess infinite collections of quadratic points are hyperelliptic and bielliptic; this conjecture was confirmed by Harris and Silverman.

I will talk about the general problem of classifying curves that possess infinite collections of degree $d$ points. I will explain how to reduce this classification problem to a study of curves of low genus, and use this reduction to obtain a classification for $d \leq 5$. This relies on analyzing a discrete-geometric object – the subspace configuration – attached to curves with infinitely many degree $d$ points. This talk is based on joint work with Isabel Vogt (arXiv:2208.01067).