Dissertation Defense

Topics in arithmetic statistics

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Abstract: Arithmetic statistics may be interpreted broadly to include questions in number theory and arithmetic geometry with a distinct quantitative flavor. To answer even simply stated such questions, we often employ diverse algebraic, analytic, or geometric techniques. This dissertation addresses several arithmetic statistical questions, and for its defense we focus on those related to superelliptic curves.

A superelliptic curve is given by an affine algebraic equation of the form $C\colon y^m = f(x)$. For a fixed such curve C and degree n, we ask how many number fields $K/\mathbb{Q}$ of degree n arise as the minimal field of definition of an algebraic point on C, as counted by discriminant? For n sufficiently large and subject to certain conditions, we find infinitely many of these fields, producing an asymptotic lower bound of the form X^δ for an explicit constant $\delta > 0$. In special cases, we are additionally able to count those extensions with prescribed Galois group.

For certain degrees n, it is possible for a curve to have only finitely many points of degree n, or even none at all. Instead of fixing a curve C, one might ask how often a curve has (or lacks) points of certain degree, as it varies in some family. In the case of superelliptic curves, we make these questions precise by counting the defining polynomials f by their coefficients. We then find that a positive proportion of superelliptic curves are everywhere locally soluble, a necessary condition for having a rational point, and pin down this proportion exactly in the trigonal genus 4 case. After placing conditions on the family, we also find that for certain degrees n, a positive proportion of curves have only finitely many points of degree n.

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