Math 421 Problem Set
September 1, 2022

1. Look back at the multiplication table you made for $(\mathbb{Z}/12\mathbb{Z})^\times$ on the last problem set. Calculate the order of each element.

2. Let G be a group and $x \in G$.

   (a) Show that $|x| = |x^{-1}|$.

   (b) If x has finite order n, show that the elements $1, x, x^2, \ldots, x^{n-1}$ are all distinct.

   (c) If x has infinite order, show that the elements x^n, $n \in \mathbb{Z}$ are all distinct.

3. Let G be a group.

   (a) For $x \in G$, show that $\{x^n \mid n \in \mathbb{Z}\}$ is a subgroup of G. This is called the ***cyclic subgroup*** of G generated by x.

   (b) Find all the cyclic subgroups of D_8.

   (c) A group G is ***cyclic*** if G is a cyclic subgroup of itself; i.e. if $G = \{x^n \mid n \in \mathbb{Z}\}$ for some $x \in G$. Check that $\mathbb{Z}/n\mathbb{Z}$ and $\mathbb{Z}$ are cyclic.

   (d) Is $(\mathbb{Z}/n\mathbb{Z})^\times$ always cyclic?