Math 421 Problem Set
September 8, 2022

1. (a) If G is a group, and $a, b \in G$ commute, show that $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$. (Do this first by induction for positive n.)

   (b) Show that the order of a cycle in S_n is equal to its length.

   (c) Show that the order of an element in S_n is equal to the least common multiple of the lengths of the cycles in its cycle decomposition.

   (d) For each element in S_3, write its cycle decomposition and compute its order.

2. Let G be a group and $H \subset G$ be some subset of G. Show that $H \leq G$ (i.e. H is a subgroup of G) if and only if H is nonempty and for all $a, b \in H$, $a^{-1} \in H$ and $ab \in H$.