

Math 421 Problem Set
September 13, 2022

1. Let $\phi : G \rightarrow G'$ be a group homomorphism.

(a) Define the *kernel* of ϕ to be

$$\ker(\phi) := \{g \in G \mid \phi(g) = 1\}.$$

Show that $\ker(\phi) \leq G$. (Hint: Use Problem 2 from the last problem set.)

(b) Show that ϕ is injective if and only if $\ker(\phi) = \{1\}$, the trivial subgroup.

(c) Show that the image of ϕ is a subgroup of G' .

(d) Show that the only homomorphism from $\mathbb{Z}/n\mathbb{Z}$ to \mathbb{Z} is the trivial homomorphism, i.e. the map sending all elements to the identity.

2. (a) Consider the groups

- D_8
- Q_8
- $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$
- $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$

Notice that each of these groups has order 8. For each pair, show that the two groups are not isomorphic.

(b) Is $S_3 \cong D_6$?