Math 421 Problem Set
September 15, 2022

1. Let H be a group acting on a set A. Consider the relation $\sim$ on A defined by $a \sim b$ if and only if $a = h \cdot b$ for some $h \in H$.

   (a) Show that $\sim$ is an equivalence relation, i.e. it's reflexive ($a \sim a$), symmetric ($a \sim b \implies b \sim a$) and transitive ($a \sim b$ and $b \sim c \implies a \sim c$). (For each $x \in A$ the equivalence class of x is called the ***orbit*** of x under the action of H. The orbits partition A).

   (b) Let G be a group and $H \leq G$ a subgroup of G. Check that H acts on G by left multiplication.

   (c) Assume G in part (b) is finite. Let $x \in G$ and let $\mathcal{O}$ be the orbit of x under the action of H. Prove that the map $f : H \to \mathcal{O}$ defined $f(h) = hx$ is a bijection.

   (d) Deduce that $|H|$ divides $|G|$. This is called *Lagrange's Theorem.*