

Math 421 Problem Set
September 20, 2022

1. Let G be an abelian group.
 - (a) Show that $\{g \in G \mid |g| < \infty\} \leq G$. This is called the *torsion subgroup* of G .
 - (b) Fix $n > 1$ a positive integer. Find the torsion subgroup of $(\mathbb{Z}/n\mathbb{Z}) \times \mathbb{Z}$.
 - (c) Show that the set of elements of $(\mathbb{Z}/n\mathbb{Z}) \times \mathbb{Z}$ of infinite order together with the identity is not a subgroup.
 - (d) **(Challenge)** Give an example to show the set in part (a) is not a subgroup if we don't require G to be abelian.

2. Let G be a group. Show that the intersection of an arbitrary (not necessarily finite) collection of subgroups of G is again a subgroup.