

**Math 421 Problem Set**  
**September 27, 2022**

1. Consider the action of  $D_8$  on itself by conjugation. That is  $g \cdot h = ghg^{-1}$  for  $g, h \in D_8$ . Find the orbits of the action, and compute the stabilizer of each element.
2. Let  $G$  be a group. An isomorphism from  $G$  to itself is called an *automorphism* of  $G$ . Let  $\text{Aut}(G)$  be the set of automorphisms of  $G$ .
  - (a) Show that  $\text{Aut}(G)$  is a group with composition as the operation.
  - (b) Show that  $\text{Aut}(G)$  acts on  $G$  by  $\phi \cdot g = \phi(g)$ , and the action has trivial kernel.
  - (c) **Challenge:** Find the orbits of the action of  $\text{Aut}(S_3)$  on  $S_3$ .