## Math 421 Problem Set October 13, 2022

- 1. Let  $H = \langle s \rangle \leq D_8$ .
  - (a) Find all the left cosets and all the right cosets of  $\langle s \rangle$  in  $D_8$ . Is it the case that gH = Hg for all  $g \in D_8$ ?
  - (b) Show that the multiplication on left cosets of H is *not* well-defined. (In particular, this means that H can't be the kernel of any homomorphism.)
- 2. Let  $H \leq G$ . Prove that the function  $f: G \to G$  defined  $f(x) = x^{-1}$  sends each left coset of H to a right coset of H and gives a bijection between the left cosets and right cosets. (Thus the number of left cosets equal the number of right cosets.)