Math 421 Problem Set
October 25, 2022

1. Let G be a group and $N \leq G$. Show that the following are equivalent.

   (i) $N \trianglelefteq G$ (i.e. $gNg^{-1} = N$ for all $g \in G$.)

   (ii) $N_G(N) = G$

   (iii) $gN = Ng$ for all $g \in G$

   (iv) $gNg^{-1} \subseteq N$ for all $g \in G$.

2. Let $\phi : G \to H$ be a homomorphism.

   (a) If $K \trianglelefteq H$, show that $\phi^{-1}(K) \trianglelefteq G$. (Recall that we already showed in a previous homework that $\phi^{-1}(K) \leq G$, so you just need to show that it is normal.)

   (b) Give an example to show that $L \trianglelefteq G$ doesn't necessarily imply $\phi(L) \trianglelefteq H$.