Math 421 Problem Set
November 1, 2022

1. Let $H \leq G$, and fix $g \in G$.
   (a) Show $gHg^{-1} \leq G$.
   (b) Show gHg^{-1} has the same order as H.
   (c) Deduce that if n is a positive integer, and H is the unique subgroup of G of order n, then $H \trianglelefteq G$.

2. Let p be a prime integer.
   (a) Find the order of the multiplicative group $(\mathbb{Z}/p\mathbb{Z})^{\times}$.
   (b) Use Lagrange's Theorem to prove Fermat's Little Theorem: $a^p \equiv a (\text{mod } p)$ for all $a \in \mathbb{Z}$. (Hint: Consider the residue class $\bar{a}$ of a in $(\mathbb{Z}/p\mathbb{Z})^{\times}$. What is $\bar{a}^{p-1}$?)