

Math 421 Problem Set
November 8, 2022

1. Let G be a group and $N \trianglelefteq G$. Recall that the Fourth Isomorphism Theorem says that there is a bijection between the subgroups of G that contain N and the subgroups of G/N . In particular, any subgroup of G/N is of the form $\bar{H} = H/N$ for some $H \leq G$ containing N .

Let A and B be subgroups of G that contain N . Prove the following.

- (a) $A \leq B$ if and only if $\bar{A} \leq \bar{B}$.
- (b) $A \trianglelefteq G$ if and only if $\bar{A} \trianglelefteq \bar{G}$.
- (c) (Challenge) If $A \leq B$, then $|B : A| = |\bar{B} : \bar{A}|$.