

Math 421 Problem Set 21
November 15, 2022

1. Let G be a group and A a nonempty set that G acts on.

- (a) Show that if $a, b \in A$ and $b = g \cdot a$ for some $g \in G$, then $G_b = gG_ag^{-1}$.
- (b) Show that if G acts transitively on A then the kernel of the action is

$$\bigcap_{g \in G} gG_ag^{-1}.$$

2. Let Q_8 act on itself by left multiplication. Use this action to find a subgroup H of S_8 that is isomorphic to Q_8 by finding its image under the corresponding permutation representation. You can just give generators for H – you don't need to list all the elements. (It might help to label the elements $1, -1, i, -i, j, -j, k, -k$ of Q_8 by $1, 2, 3, 4, 5, 6, 7, 8$, respectively, like we did in class.)