# Math 421 Problem Set 22
# November 17, 2022

1. Let $G$ be a non-cyclic group of order 6.

   (a) Show that each nontrivial element of $G$ has order 2 or 3.

   (b) Show that the nontrivial elements can't all have the same order, and thus $G$ has an element $x$ of order 2 and $y$ of order 3.

   (c) Show that if $xy = yx$, then $G = \langle xy \rangle$. Conclude that $xy \neq yx$.

   (d) Use part (c) to show that $\langle x \rangle$ is not normal.

   (e) Consider the action by left multiplication of $G$ on the set of left cosets $A$ of $\langle x \rangle$. Let $\pi_H : G \to S_A$ be the associated permutation representation. Show that $\ker \pi_H = 1$. (It might help to use a theorem from class.)

   (f) Conclude that the only two groups of order 6 (up to isomorphism) are $Z_6$ and $S_3$.