Math 421 - Quiz 12 Name: solutions
December 1, 2022

1. Let $G = Z_2 \times Z_2 = \langle a, b \mid a^2 = b^2 = 1, ab = ba \rangle$. Let G act on itself by left multiplication.

   (a) [*2 points*] Compute $a \cdot g$ and $b \cdot g$ for each element $g \in G$.

   $G = \{1, a, b, ab\}$

$a \cdot 1 = a$	$b \cdot 1 = b$
$a \cdot a = 1$	$b \cdot a = ab$
$a \cdot b = ab$	$b \cdot b = 1$
$a \cdot ab = b$	$b \cdot ab = a$

   (b) [*2 points*] By labeling the elements of G from 1 through 4 like we did in class, describe the corresponding permutation representation $\phi : G \to S_4$ by giving the images of the generators.

   $1 \leftrightarrow 1 \quad a \leftrightarrow 2 \quad b \leftrightarrow 3 \quad ab \leftrightarrow 4$

   $\phi(a) = (1\ 2)(3\ 4)$
   $\phi(b) = (1\ 3)(2\ 4)$

   (c) [*1 point*] Using your answer from part (b), give generators for a subgroup of S_4 that is isomorphic to G.

   $\langle (1\ 2)(3\ 4), (1\ 3)(2\ 4) \rangle$