Math 421 - Quiz 1
September 1, 2022

Name: Solutions

1. [*2 points*] Complete the following definition:

A *group* is a set G along with a binary operation $*$ on G which satisfies the following axioms:

(a) Associativity: $\forall\, a,b,c \in G$, $(a*b)*c = a*(b*c)$

(b) $\exists\, e \in G$ such that $e*g = g*e$ $\forall\, g \in G$.

(c) $\forall\, g \in G$, $\exists\, g^{-1} \in G$ s.t. $g*g^{-1} = e = g^{-1}*g$

2. [*2 points*] Let G be the set of all functions from $\mathbb{R}$ to $\mathbb{R}$. Either show that G, along with the binary operation of composition (denoted $\circ$), is a group or give a counterexample to show that it is not a group.

G is n<u>o</u>t a group.
Notice that $f \in G$ defined $f(x) = x$ is an identity element. However, the function $g(x) = x^2$ is not bijective and therefore has no inverse under composition.

3. [*1 point*] Define $*$ by $a*b = \sqrt{a+b}$. Is $*$ a binary operation on $\mathbb{Z}$? Why or why not?

No: $1*1 = \sqrt{1+1} = \sqrt{2}$, which is not an integer.