

Math 421 - Quiz 2  
September 8, 2022

Name: Solutions

1. [2 points] Consider the group  $D_8$ , which has underlying set  $\{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$ .

(a) What is  $(sr^2)(sr^3)$ ? Your answer should be one of the above elements. No explanation necessary.

$$(sr^2)(sr^3) = \underbrace{(sr^2)(sr^2)}_1 r = r$$

(b) Is  $D_8$  cyclic? Briefly explain.

No. All elements have order  $< 8$ .

2. [3 points] Let  $G$  be a group and  $x \in G$ . Recall from the homework that  $(x^a)^b = x^{ab}$  and  $x^a x^b = x^{a+b}$  for all integers  $a$  and  $b$ . Show that  $x$  and  $x^{-1}$  have the same order.

$$\S (x^{-1})^n = 1$$

$$\Leftrightarrow x^{-n} = 1$$

$$\Leftrightarrow (x^n)^{-1} = 1$$

$$\Leftrightarrow x^n = 1$$

Thus,  $|x^{-1}| = |x|$ .