Math 421 - Quiz 3
September 15, 2022

Name: Solutions

1. [*2 points*] Write each of the following elements of S_6 as a product of disjoint cycles. No justification necessary.

   (a) σ defined $\sigma(1) = 1, \sigma(2) = 6, \sigma(3) = 2, \sigma(4) = 5, \sigma(5) = 4, \sigma(6) = 3$.

   $$\sigma = (2\,6\,3)(4\,5)$$

   (b) $\tau = (26)(123)$.

   $$\tau = (1\,6\,2\,3)$$

2. [*3 points*] Let G be a group and $H \subset G$ some **nonempty** subset. Show that $ab^{-1} \in H$ for all $a, b \in H$ if and only if H is a subgroup of G.

If $H \leq G$, then $\forall\, a, b \in H$, $ab^{-1} \in H$ since $b^{-1} \in H$ and H is closed under multiplication.

For the converse, assume $H \subseteq G$ and $ab^{-1} \in H$ $\forall\, a, b \in H$.

Let $a \in H$. Then $1 = aa^{-1} \in H$ and $a^{-1} = 1a^{-1} \in H$, so H has an identity and inverses.

If $a, b \in H$, then $b^{-1} \in H$ so $ab = a(b^{-1})^{-1} \in H$, so H is closed under the operation.

The operation is associative on G, so it must be on H as well.