Math 421 - Quiz 4 **Name:** Solutions
September 22, 2022

1. [*2 points*] Let G act on itself by left multiplication. That is, the action is defined by $g \cdot h = gh$ for $g, h \in G$. Is this action faithful? Briefly explain.

Yes: Suppose $g \in G$ acts as the identity. i.e. $g \cdot h = h$ $\forall$ $h \in G$. Then

$$gh = h \Rightarrow g = 1.$$

Thus, 1 is the only element that fixes every element, so the kernel of $G \to S_G$ is 1.

2. [*3 points*] Let G and H be groups and $\phi : G \to H$ an isomorphism. Show that the inverse function ϕ^{-1} is a homomorphism (and thus an isomorphism as well).

Let $a, b \in H$. Then

$$a = \varphi(a'), \quad b = \varphi(b')$$

for some $a', b' \in G$, since φ is surjective.

$$\begin{aligned} \Rightarrow \varphi^{-1}(ab) &= \varphi^{-1}(\varphi(a')\varphi(b')) \\ &= \varphi^{-1}(\varphi(a'b')) \\ &= a'b' \\ &= \varphi^{-1}(a)\varphi^{-1}(b). \end{aligned}$$