Math 421 - Quiz 5
September 29, 2022

Name: Solutions

1. [*1 point*] Consider S_n along with its natural action on the set $\{1, 2, \ldots, n\}$. Describe the stabilizer of n. No explanation needed.

The stabilizer is the set of elements of S_n whose cycle decompositions don't contain n. It's naturally isomorphic to S_{n-1}.

2. [*1 point*] What is the torsion subgroup of $\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$? No explanation needed.

The torsion subgroup is

$$\{(0,0),(0,1)\} = \{0\} \times \mathbb{Z}/2\mathbb{Z} \cong \mathbb{Z}/2\mathbb{Z}$$

3. [*3 points*] Let $H \leq G$ be a proper subgroup (i.e. $H \neq G$). Show that G does **not** act on H by left multiplication.

Since H is proper, we can take $g \in G \setminus H$.

Then $1 \in H$, but $g \cdot 1 = g \notin H$.

Thus, left multiplication does not give a function $G \times H \to H$, so it's not a group action.