

Math 421 - Quiz 6
October 6, 2022

Name: solutions

1. [2 points] How many subgroups does Z_{18} (the multiplicative cyclic group with 18 elements, $1, x, x^2, \dots, x^{17}$) have and what are their orders? No explanation needed.

Orders: 1, 2, 3, 6, 9, 18 (factors of 18)

6 total subgroups

2. [1 point] If Z_{20} is generated by x , what is the order of x^{15} ? No explanation needed.

$$|x^{15}| = \frac{20}{\text{g.c.d.}(20, 15)} = \frac{20}{5} = 4$$

3. [2 points] Show that if a group G has prime order p , it must be cyclic.

Let $x \in G$ where $x \neq 1$.

Then $\langle x \rangle$ is a subgroup of order $|x|$.

Thus $|x| \mid p$ and $|x| \neq 1$, so

$|x| = p$. Thus $G = \langle x \rangle$, so G is cyclic.