

Math 421 - Quiz 8
October 27, 2022

Name: solutions

1. Let $G = \mathbb{Z}/8\mathbb{Z} = \{0, 1, \dots, 7\}$ and $K = \langle 4 \rangle \leq G$.

(a) [2 points] Find all the left cosets of K in G . (Keep in mind the operation is addition.)

$$0 + K = \{0, 4\}$$

$$1 + K = \{1, 5\}$$

$$2 + K = \{2, 6\}$$

$$3 + K = \{3, 7\}$$

(b) [2 points] Explicitly define a homomorphism $\phi: G \rightarrow \mathbb{Z}/4\mathbb{Z}$ such that $K = \ker \phi$.

$$\phi(x) = x \pmod{4}$$

More explicitly,

$$\phi(0 + K) = 0$$

$$\phi(1 + K) = 1$$

$$\phi(2 + K) = 2$$

$$\phi(3 + K) = 3$$

(c) [1 points] To what familiar group is G/K isomorphic?

Since ϕ is surjective, $G/K \cong \mathbb{Z}/4\mathbb{Z}$