

1. [1 point] Give an example of a subgroup of a group that is not normal. No explanation necessary.

$$\langle (12) \rangle \leq S_3 \text{ is not normal}$$

$$\left((13)(12)(13)^{-1} = (23) \notin \langle (12) \rangle \right)$$

2. [1 point] Let $H = \langle (123) \rangle \leq S_3$. What is the order of S_3/H ? No explanation necessary.

$$|S_3/H| = \frac{|S_3|}{|H|} = \frac{6}{3} = 2$$

3. [3 points] Let G be a group and $N \trianglelefteq G$. Let $x \in G$ and $xN \in G/N$. Suppose that $|x| = n$. Show that $|xN| \leq n$.

$$(xN)^n = x^n N = 1N = 1 \in G/N$$

Thus, $|xN| \leq n$.