Subgroups

Let G be a group. Recall that before, we defined a subgroup H in G to be a subset of G s.t. H is a group under the binary operation of G.

However, if we know G is a group, we don't actually need to check all The group axioms for H:

Pf: $|f H \leq G_1$ then (1) holds since |f H | and (2) holds since $y^{-1} \in H$ and H is closed under multiplication.

For the converse, assume H is a subset of G that satisfies (1) and (2).

Let
$$a \in H$$
 (by (1)). Then $|=aa^{-1} \in H$ and $a^{-1} = |a^{-1} \in |H|$
by (2), so H has an identity and inverses.

If $a, b \in H$, Then $b^{-1} \in H$, so $ab = a(b^{-1})^{-1} \in H$, so H is closed under the operation. The operation is associative on G, so it must be on H as well. []

Ex:
1.) Every group G has
$$E^{13} \leq G$$
 and $G \leq G$. E^{13} is
called the trivial subgroup, and is denoted just 1.

2.)
$$\{1,r,r^2,...,r^n\} \in D_{2n} \text{ and } \{1,s\} \in D_{2n}.$$

3.)
$$S_k \leq S_n$$
 for $k \leq n$.

4.) If $K \in H$ and $H \in G$, then $K \in G$.