

Hints for HW of Chapter 10.

Math 463/663
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10.1. Justify yourself that $x = Py$, P is a permutation matrix.

(1)

10.2. Assume that x, y, z are all ordered in nonincreasing order.

(a) Set $d_i = y_i - x_i$.

$$\begin{aligned} \sum_{i=1}^m (y_i - x_i) z_i &= \sum_{i=1}^m d_i z_i \\ &= d_1(z_1 - z_2) \geq 0 \\ &\quad + (d_1 + d_2)(z_2 - z_3) \geq 0 \\ &\quad + (d_1 + d_2 + d_3)(z_3 - z_4) \geq 0 \\ &\quad \vdots \\ &\quad + (d_1 + \dots + d_{m-1})(z_{m-1} - z_m) \geq 0 \\ &\quad + \underbrace{(d_1 + \dots + d_m)}_0 z_m \\ &\geq 0. \end{aligned}$$

(b) Mimic ^{to} proof for (a).

10.3 Verify this by definition 10.1.

(2)

$$10.5. \quad x \prec y \Leftrightarrow \sum_{i=1}^m |x_i - a| \leq \sum_{i=1}^m |y_i - a| \text{ for all } a \in \mathbb{R}.$$

① " \Rightarrow ". Thm 10.9 (Easy part).

② " \Leftarrow ".

First by choosing a very large or very small, we can show that

$$\sum_{i=1}^m x_i = \sum_{i=1}^m y_i.$$

Secondly, notice that $|x_i - a| = 2(x_i - a)_+ - (x_i - a)$

$$|y_i - a| = 2(y_i - a)_+ - (y_i - a)$$

where $(x)_+ = \max(x, 0)$. Choose $a = y_{[k]}$

10.6. You need to show that $P 1_m = P' 1_m = 1_m$.

$$1_m \prec P 1_m \prec 1_m$$

1_m is the smallest element.

assumption on P .

\Rightarrow Ex 10.1.

\exists a permutation matrix Q s.t.

$$1_m = Q P 1_m.$$

$$\text{or } P 1_m = 1_m.$$

How about $P' 1_m = 1_m$??

