Hints for HW 4 Chapter 6.

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6.4. (a) By Thm 5.3 (i),

$$A^{+} = A'(AA')^{-1} = \frac{1}{40}\begin{pmatrix} 0 & 10 \\ 4 & 3 \\ -12 & 11 \\ -4 & 7 \\ 12 & -1 \end{pmatrix}$$

Then use Thm 6.2 to check consistency.

(b) Use the formula (6.3) on p. 251 to give the general solution.

(c) Apply Thm 6.7 to find the number of linearly indep. sol.

(d) Follow example 6.6.

6.6. $A^{+} = A'(AA')^{-1} = \frac{1}{54}\begin{pmatrix} -16 & 23 \\ 22 & -8 \\ 4 & 1 \end{pmatrix}$, $B^{+} = (B'B)^{-1}B' = \frac{1}{3}\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

(a) Thm 6.3.

(b). Thm 6.5.

6.5. Mimic the proof of theorem 6.4.

6.12. $Ax_* = c$, $A'y_* = d$. $\Rightarrow$ $d'x_* = \cdots = \cdots = c'y_*$.

replace d by $A'y_*$

6.13. Mimic the proof of thm 6.16.

First, since the system is consistent, write the general sol. as

$$x_* = A^-c + (I_n - A^-A)y$$

for any y. (see theorem 6.4).

You will need to show that $(I_n - A^-A)A^-c = 0$

where you need to use the fact that the system is consistent, i.e. (by theorem 6.2) $AA^-c = c$.

6.20.

(a) A^+ is a special case of A^L. So one may find A^+ here

First comput. $A'A$, which is symmetric.

Then find $(A'A)^+$ using spectral decomposition (follow Example 5.2 on p205)

Then $A^+ = (A'A)^+A'$ (Following Thm 5.3 (g))

(b) Thm 6.2

(c) Thm 6.14

(d) Thm 6.15.

6.21. $x_* = A^L c \iff A'A x_* = A'c.$

"$\Rightarrow$" $\quad x_* = A^L c \Rightarrow A'A x_* = A'AA^L c \overset{(5.3)}{=} A'(AA^L)'c = \cdots = A'c.$

"$\Leftarrow$"

$$A'A x_* = A'c \Rightarrow A^{L'}A'A x_* = A^{L'}A'c$$

$$A^{L'}A'A x_* = (AA^L)'A x_* \overset{(5.3)}{=} AA^L A x_* \overset{(5.1)}{=} A x_*$$

$$A^{L'}A'c = (AA^L)'c \overset{(5.3)}{=} AA^L c$$

Then Apply Thm 6.13.