over 100 million matrix multiplications per second. This is particularly important in the field of three-dimensional graphics where the homogeneous coordinates have four components and 4×4 matrices are required.

Exercises for 4.5

Exercise 4.5.1 Consider the letter A described in Figure 4.5.2. Find the data matrix for the letter obtained by:

a. Rotating the letter through $\frac{\pi}{4}$ about the origin.

b. Rotating the letter through $\frac{\pi}{4}$ about the point $\left[\begin{array}{c} 1 \\ 2 \end{array} \right]$.

Exercise 4.5.2 Find the matrix for turning the letter A in Figure 4.5.2 upside-down in place.

Exercise 4.5.3 Find the 3×3 matrix for reflecting in the line $y = mx + b$. Use $\left[\begin{array}{c} 1 \\ m \end{array} \right]$ as direction vector for the line.

Exercise 4.5.4 Find the 3×3 matrix for rotating through the angle θ about the point $P(a, b)$.

Exercise 4.5.5 Find the reflection of the point P in the line $y = 1 + 2x$ in $\mathbb{R}^2$ if:

a. $P = P(1, 1)$

b. $P = P(1, 4)$

c. What about $P = P(1, 3)$? Explain. [*Hint*: Example 4.5.1 and Section 4.4.]

Supplementary Exercises for Chapter 4

Exercise 4.1 Suppose that $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors. If $\mathbf{u}$ and $\mathbf{v}$ are not parallel, and $a\mathbf{u} + b\mathbf{v} = a_1\mathbf{u} + b_1\mathbf{v}$, show that $a = a_1$ and $b = b_1$.

Exercise 4.2 Consider a triangle with vertices A, B, and C. Let E and F be the midpoints of sides AB and AC, respectively, and let the medians EC and FB meet at O. Write $\overrightarrow{EO} = s\overrightarrow{EC}$ and $\overrightarrow{FO} = t\overrightarrow{FB}$, where s and t are scalars. Show that $s = t = \frac{1}{3}$ by expressing $\overrightarrow{AO}$ two ways in the form $a\overrightarrow{EO} + b\overrightarrow{AC}$, and applying Exercise 4.1. Conclude that the medians of a triangle meet at the point on each that is one-third of the way from the midpoint to the vertex (and so are concurrent).

Exercise 4.3 A river flows at 1 km/h and a swimmer moves at 2 km/h (relative to the water). At what angle must he swim to go straight across? What is his resulting speed?

Exercise 4.4 A wind is blowing from the south at 75 knots, and an airplane flies heading east at 100 knots. Find the resulting velocity of the airplane.

Exercise 4.5 An airplane pilot flies at 300 km/h in a direction 30° south of east. The wind is blowing from the south at 150 km/h.

a. Find the resulting direction and speed of the airplane.

b. Find the speed of the airplane if the wind is from the west (at 150 km/h).

Exercise 4.6 A rescue boat has a top speed of 13 knots. The captain wants to go due east as fast as possible in water with a current of 5 knots due south. Find the velocity vector $\mathbf{v} = (x, y)$ that she must achieve, assuming the x and y axes point east and north, respectively, and find her resulting speed.

Exercise 4.7 A boat goes 12 knots heading north. The current is 5 knots from the west. In what direction does the boat actually move and at what speed?

Exercise 4.8 Show that the distance from a point A (with vector $\mathbf{a}$) to the plane with vector equation $\mathbf{n} \cdot \mathbf{p} = d$ is $\frac{1}{\|n\|}|\mathbf{n} \cdot \mathbf{a} - d|$.

Exercise 4.9 If two distinct points lie in a plane, show that the line through these points is contained in the plane.

Exercise 4.10 The line through a vertex of a triangle, perpendicular to the opposite side, is called an **altitude** of the triangle. Show that the three altitudes of any triangle are concurrent. (The intersection of the altitudes is called the **orthocentre** of the triangle.) [*Hint*: If P is the intersection of two of the altitudes, show that the line through P and the remaining vertex is perpendicular to the remaining side.]