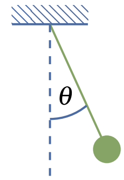
Exercise 6.6.8 Consider a spring, as in Example 6.6.4. If the period of the oscillation is 30 seconds, find the spring constant k.

Exercise 6.6.9 As a pendulum swings (see the diagram), let t measure the time since it was vertical. The angle $\theta = \theta(t)$ from the vertical can be shown to satisfy the equation $\theta'' + k\theta = 0$, provided that θ is small. If the maximal angle is $\theta = 0.05$ radians, find $\theta(t)$ in terms of k. If the period is 0.5 seconds, find k. [Assume that $\theta = 0$ when $t = 0$.]



Supplementary Exercises for Chapter 6

Exercise 6.1 (Requires calculus) Let V denote the space of all functions $f : \mathbb{R} \to \mathbb{R}$ for which the derivatives f' and f'' exist. Show that f_1, f_2, and f_3 in V are linearly independent provided that their **wronskian** $w(x)$ is nonzero for some x, where

$$w(x) = \det \begin{bmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_1'(x) & f_2'(x) & f_3'(x) \\ f_1''(x) & f_2''(x) & f_3''(x) \end{bmatrix}$$

Exercise 6.2 Let $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ be a basis of $\mathbb{R}^n$ (written as columns), and let A be an $n \times n$ matrix.

a. If A is invertible, show that $\{A\mathbf{v}_1, A\mathbf{v}_2, \dots, A\mathbf{v}_n\}$ is a basis of $\mathbb{R}^n$.

b. If $\{A\mathbf{v}_1, A\mathbf{v}_2, \dots, A\mathbf{v}_n\}$ is a basis of $\mathbb{R}^n$, show that A is invertible.

Exercise 6.3 If A is an $m \times n$ matrix, show that A has rank m if and only if col A contains every column of I_m.

Exercise 6.4 Show that null A = null (A^TA) for any real matrix A.

Exercise 6.5 Let A be an $m \times n$ matrix of rank r. Show that $\dim(\text{null } A) = n - r$ (Theorem 5.4.3) as follows. Choose a basis $\{\mathbf{x}_1, \dots, \mathbf{x}_k\}$ of null A and extend it to a basis $\{\mathbf{x}_1, \dots, \mathbf{x}_k, \mathbf{z}_1, \dots, \mathbf{z}_m\}$ of $\mathbb{R}^n$. Show that $\{A\mathbf{z}_1, \dots, A\mathbf{z}_m\}$ is a basis of col A.