

Math 221: LINEAR ALGEBRA

Chapter 1. Systems of Linear Equations §1-5. Application to Electrical Networks

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
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¹Slides are adapted from those by Karen Seyffarth from University of Calgary.

Resistor Networks

Important Symbols:

Resistor: 

Voltage Source: 


Current:



Resistor Networks

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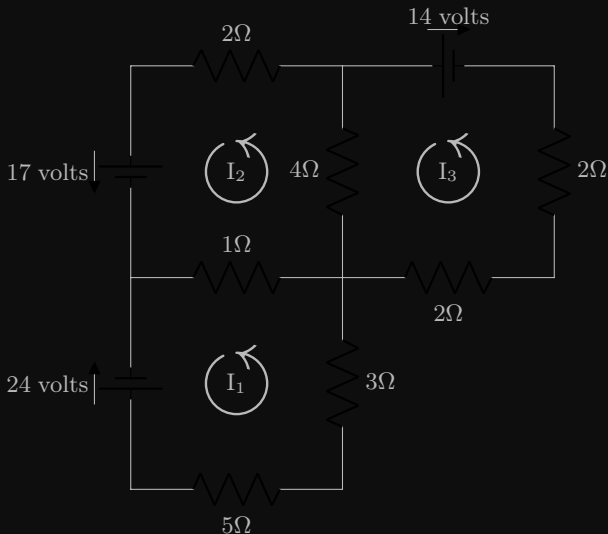
Current:



Resistance is measured in ohms, Ω . Voltage is measured in volts, V. Current is measured in amps, A.

Problem

Write an equation for each circuit and solve for each current in the following diagram.



Solution

The equation for the bottom circuit, with current I_1 is given by

$$5I_1 + 3I_1 + I_1 - I_2 = -24$$

The top left circuit, with current I_2 is

$$I_2 - I_1 + 4I_2 - 4I_3 + 2I_2 = 17$$

Solution

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The top left circuit, with current I_2 is

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The top right circuit is

$$4I_3 - 4I_2 + 2I_3 + 2I_3 = -14$$

Solution

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The top right circuit is

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After simplifying, this system is represented by

$$\left[\begin{array}{ccc|c} 9 & -1 & 0 & -24 \\ -1 & 7 & -4 & 17 \\ 0 & -4 & 8 & -14 \end{array} \right]$$

Solution (continued)

The reduced row-echelon form of this matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{5}{3} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & -1 \end{array} \right]$$

Solution (continued)

The reduced row-echelon form of this matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & -1 \end{array} \right]$$

This gives values of the currents of

$$I_1 = -\frac{5}{2}$$

$$I_2 = \frac{3}{2}$$

$$I_3 = -1$$