

Math 221: LINEAR ALGEBRA

Chapter 1. Systems of Linear Equations §1-6. Application to Chemical Reactions

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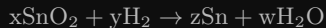
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¹Slides are adapted from those by Karen Seyffarth from University of Calgary.

Balancing Chemical Reactions

Problem

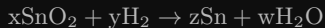
Balance the chemical reaction given below involving tin (Sn), hydrogen (H), and oxygen (O).



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Solution

Setting up a system of equations in x, y, z, w gives

$$\text{Sn} \quad : \quad x = z \text{ or } x - z = 0$$

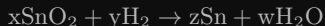
$$\text{O} \quad : \quad 2x = w \text{ or } 2x - w = 0$$

$$\text{H} \quad : \quad 2y = 2w \text{ or } 2y - 2w = 0$$

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$$\text{H} \quad : \quad 2y = 2w \text{ or } 2y - 2w = 0$$

The augmented matrix is
$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 \end{array} \right]$$

Solution (continued)

The reduced row-echelon matrix is

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array} \right]$$

Letting $w = t$, the solution is

$$x = \frac{1}{2}t$$

$$y = t$$

$$z = \frac{1}{2}t$$

$$w = t$$

Solution (continued)

The reduced row-echelon matrix is

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Letting $w = t$, the solution is

$$\begin{aligned} x &= \frac{1}{2}t \\ y &= t \\ z &= \frac{1}{2}t \\ w &= t \end{aligned}$$

We can choose any values for $w = t$. Suppose we choose $w = 4$, then $x = 2, y = 4, z = 2$ and the balanced reaction is

