Math 221: LINEAR ALGEBRA

Chapter 1. Systems of Linear Equations

§1-6. Application to Chemical Reactions

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Emory University, 2020 Fall

(last updated on 10/26/2020)

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[1] Slides are adapted from those by Karen Seyffarth from University of Calgary.

Balancing Chemical Reactions

Problem

Balance the chemical reaction given below involving tin (Sn), hydrogen (H), and oxygen (0).

$$\mathrm{xSnO_2 + yH_2 \rightarrow zSn + wH_2O}$$

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Solution

Setting up a system of equations in $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{w}$ gives

$$\begin{aligned} \mathrm{Sn} &: \quad \mathrm{x} = \mathrm{z} \text{ or } \mathrm{x} - \mathrm{z} = 0 \\ \mathrm{O} &: \quad 2\mathrm{x} = \mathrm{w} \text{ or } 2\mathrm{x} - \mathrm{w} = 0 \\ \mathrm{H} &: \quad 2\mathrm{y} = 2\mathrm{w} \text{ or } 2\mathrm{y} - 2\mathrm{w} = 0 \end{aligned}$$

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The augmented matrix is $\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 \end{array}\right]$

Solution (continued)

The reduced row-echelon matrix is

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array}\right]$$

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Letting $w = t$, the solution is

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We can choose any values for $w = t$. Suppose we choose $w = 4$, then $x = 2, y = 4, z = 2$ and the balanced reaction is

$$2Sn0_2 + 4H_2 \rightarrow 2Sn + 4H_2O$$