
Homework 2

Due on Jan. 28

§5.5: 1, 2, 3, 4.

§5.3: 1, 4, 10, 14, 20, 24, 26, 28.

§5.5: Minimum-Variance Estimators: The Cramér-Rao Lower Bound

Question 5.5.1 Let $Y_1, Y_2, \dots, Y_n$ be a random sample from $f_Y(y; \theta) = \frac{1}{\theta} e^{-y/\theta}$, $y > 0$. Compare the Cramér-Rao lower bound for $f_Y(y; \theta)$ to the variance of the maximum likelihood estimator for θ , $\hat{\theta} = \frac{1}{n} \sum_{i=1}^n Y_i$. Is $\hat{\theta}$ a best estimator for θ ?

Question 5.5.2 Let $X_1, \dots, X_n$ be a random sample of size n from the Poisson distribution,

$$p_X(k; \lambda) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, \dots$$

Show that $\hat{\lambda} = \frac{1}{n} \sum_{i=1}^n X_i$ is an efficient estimator for λ .

Question 5.5.3 Suppose a random sample of size n is taken from a normal distribution with mean μ and variance σ^2 , where σ^2 is known. Compare the Cramér-Rao lower bound for $f_Y(y; \mu)$ with the variance of $\hat{\mu} = \bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$. Is $\bar{Y}$ efficient estimator for μ ?

Question 5.5.4 Let $Y_1, \dots, Y_n$ be a random sample from the uniform pdf $f_Y(y; \theta) = 1/\theta$ for $y \in [0, \theta]$. Compare the Cramér-Rao lower bound for $f_Y(y; \theta)$ with the variance of the unbiased estimator $\hat{\theta} = \frac{n+1}{n} Y_{\max}$. Discuss.

Question 5.3.1 A commonly used IQ test is scaled to have a mean of 100 and a standard deviation of $\sigma = 15$. A school counselor was curious about the average IQ of the students in her school and took a random sample of fifty students' IQ scores. The average of these was $y = 107.9$. Find a 95% confidence interval for the student IQ in the school.

Question 5.3.4 A physician who has a group of thirty-eight female patients aged 18 to 24 on a special diet wishes to estimate the effect of the diet on total serum cholesterol. For this group, their average serum cholesterol is 188.4 (measured in mg/100mL). Because of a large-scale government study, the physician is willing to assume that the total serum cholesterol measurements are normally distributed with standard deviation of $\sigma = 40.7$. Find a 95% confidence interval of the mean serum cholesterol of patients on the special diet. Does the diet seem to have any effect on their serum cholesterol, given that the national

average for women aged 18 to 24 is 192.0?

Question 5.3.10 In 1927, the year he hit sixty home runs, Babe Ruth batted .356, having collected 192 hits in 540 official at-bats (140). Based on his performance that season, construct a 95% confidence interval for Ruth's probability of getting a hit in a future at-bat.

Question 5.3.14 If $(0.57, 0.63)$ is a 50% confidence interval for p , what does k/n equal, and how many observations were taken?

Question 5.3.20 Viral infections contracted early during a woman's pregnancy can be very harmful to the fetus. One study found a total of 86 deaths and birth defects among 202 pregnancies complicated by a first-trimester German measles infection (45). Is it believable that the true proportion of abnormal births under similar circumstances could be as high as 50%? Answer the question by calculating the margin of error for the sample proportion, $86/202$.

Question 5.3.24 Given that a political poll shows that 52% of the sample favors Candidate A, whereas 48% would vote for Candidate B, and given that the margin of error associated with the survey is 0.05, does it make sense to claim that the two candidates are tied? Explain.

Question 5.3.26 Suppose that p is to be estimated by X/n and we are willing to assume that the true p will not be greater than 0.4. What is the smallest n for which X/n will have a 99% probability of being within 0.05 of p ?

Question 5.3.28 University officials are planning to audit 1586 new appointments to estimate the proportion p who have been incorrectly processed by the payroll department.
(a) How large does the sample size need to be in order for X/n , the sample proportion, to have an 85% chance of lying within 0.03 of p ?
(b) Past audits suggest that p will not be larger than 0.10. Using that information, recalculate the sample size asked for in part (a).