**Homework 6** Due on March 3

§7.4: 1, 2, 5, 7, 10, 12, 19, 21.
§7.5: 1, 2, 3, 6, 8, 14, 16.

## §7.4: Drawing Inferences About $\mu$

**Question 7.4.1** Use Appendix Table A.2 to find the following probabilities:
(a) $\mathbb{P}(T_6 \geqslant 1.134)$.
(b) $\mathbb{P}(T_{15} \leqslant 0.866)$.
(c) $\mathbb{P}(T_3 \geqslant -1.250)$.
(d) $\mathbb{P}(-1.055 < T_{29} < 2.462)$.

**Question 7.4.2** What values of x satisfy the following equations?
(a): $\mathbb{P}(-x \leqslant T_{22} \leqslant x) = 0.98$.
(b): $\mathbb{P}(T_{13} \geqslant x) = 0.85$.
(c): $\mathbb{P}(T_{26} < x) = 0.95$.
(d): $\mathbb{P}(T_2 \geqslant x) = 0.025$.

**Question 7.4.5** Suppose a random sample of size $n = 11$ is drawn from a normal distribution with $\mu = 15.0$. For what value of $k$ is the following true?

$$\mathbb{P}\left(\left|\frac{\overline{Y} - 15.0}{S/\sqrt{11}}\right| \geqslant k\right) = 0.05.$$

**Question 7.4.7** Cell phones emit radio frequency energy that is absorbed by the body when the phone is next to the ear and may be harmful. The table in the next column gives the absorption rate for a random sample of twenty cell phones. (The Federal Communication Commission sets a maximum of 1.6 watts per kilogram for the absorption rate of such energy.) Construct a 90% confidence interval for the true average cell phone absorption

rate.

| | |
|---|---|
| 1.54 | 1.41 |
| 1.54 | 1.40 |
| 1.49 | 1.40 |
| 1.49 | 1.39 |
| 1.48 | 1.39 |
| 1.45 | 1.39 |
| 1.44 | 1.38 |
| 1.42 | 1.38 |
| 1.41 | 1.37 |
| 1.41 | 1.33 |

*Hint:* $\sum_{i=1}^{20} y_i = 28.51$ and $\sum_{i=1}^{20} y_i^2 = 40.70$.

**Question 7.4.10** How long does it take to fly from Atlanta to New York's LaGuardia airport? There are many components of the time elapsed, but one of the more stable measurements is the actual in-air time. For a sample of 83 flights between these destinations on Sundays in April, the time in minutes (y) gave the following results:

$$\sum_{i=1}^{83} y_i = 8622 \quad \text{and} \quad \sum_{i=1}^{83} y_i^2 = 899,750.$$

Find a 99% confidence interval for the average flight time.

**Question 7.4.12** If a normally distributed sample of size $n = 16$ produces a 95% confidence interval for $\mu$ that reanges from 44.7 to 49.9, what are the values of $\bar{y}$ and $s$?

**Question 7.4.19** MBAs R Us advertises that its program increases a person's score on the GMAT by an average of forty points. As a way of checking the validity of that claim, a consumer watchdog group hired fifteen students to take both the review course and the GMAT. Prior to starting the course, the fifteen students were given a diagnostic test that predicted how well they would do on the GMAT in the absence of any special training. The following table gives each student's actual GMAT score minus his or her predicted score. Set up and carry out an appropriate hypothesis test. Use the 0.05 level of significance.

| Subject | $y_i$ = act. GMAT − pre. GMAT | $y_i^2$ |
|---|---|---|
| SA | 35 | 1225 |
| LG | 37 | 1369 |
| SH | 33 | 1089 |
| KN | 34 | 1156 |
| DF | 38 | 1444 |
| SH | 40 | 1600 |
| ML | 35 | 1225 |
| JG | 36 | 1296 |
| KH | 38 | 1444 |
| HS | 33 | 1089 |
| LL | 28 | 784 |
| CE | 34 | 1156 |
| KK | 47 | 2209 |
| CW | 42 | 1764 |
| DP | 46 | 2116 |

**Question 7.4.21** A manufacturer of pipe for laying underground electrical cables is concerned about the pipe's rate of corrosion and whether a special coating may retard that rate. As a way of measuring corrosion, the manufacturer examines a short length of pipe and records the depth of the maximum pit. The manufacturer's tests have shown that in a year's time in the particular kind of soil the manufacturer must deal with, the average depth of the maximum pit in a foot of pipe is 0.0042 inch. To see whether that average can be reduced, ten pipes are coated with a new plastic and buried in the same soil. After one year, the following maximum pit depths are recorded (in inches): 0.0039, 0.0041, 0.0038, 0.0044, 0.0040, 0.0036, 0.0034, 0.0036, 0.0046, and 0.0036. Given that the sample standard deviation for these ten measurements is 0.00383 inch, can it be concluded at the $\alpha = 0.05$ level of significance that the plastic coating is beneficial?

**Question 7.5.1** Use Appendix Table A.3 to find the following cutoffs and indicate their location on the graph of the appropriate chi square distribution.
(a) $\chi^2_{0.95,14}$
(b) $\chi^2_{0.90,2}$
(c) $\chi^2_{0.025,9}$

**Question 7.5.2** Evaluate the follwoing probabilities
(a) $\mathbb{P}\left(\chi^2_{17} \geqslant 8.672\right)$
(b) $\mathbb{P}\left(\chi^2_{6} < 10.645\right)$
(c) $\mathbb{P}\left(9.591 \leqslant \chi^2_{20} \leqslant 34.170\right)$
(d) $\mathbb{P}\left(\chi^2_{2} < 9.210\right)$

**Question 7.5.3** Find the value $y$:
(a) $\mathbb{P}\left(\chi^2_{9} \geqslant y\right) = 0.99$
(b) $\mathbb{P}\left(\chi^2_{15} \leqslant y\right) = 0.05$
(c) $\mathbb{P}\left(9.542 \leqslant \chi^2_{22} \leqslant y\right) = 0.09$

(d) $\mathbb{P}\left(y \leqslant \chi^2_{31} \leqslant 48.232\right) = 0.95$

**Question 7.5.6** Let $Y_1, Y_2, ..., Y_n$ be a random sample of size n from a normal distribution having mean $\mu$ and variance $\sigma^2$ . What is the smallest value of $n$ for which the following is true

$$\mathbb{P}\left(\frac{S^2}{\sigma^2} < 2\right) \geqslant 0.95.$$

*Hint:Use a trial-and-error method or run a R code to find the number.*

**Question 7.5.8** A random sample of size $n = 19$ is drawn from a normal distribution for which $\sigma^2 = 12.0$. In what range are we likely to find the sample variance, $s^2$? Answer the question by finding two numbers $a$ and $b$ such that

$$\mathbb{P}\left(a \leqslant S^2 \leqslant b\right) = 0.95.$$

**Question 7.5.14** Let $Y_1, \cdots, Y_n$ be a random sample of size $n$ from the pdf

$$f_Y(y) = \frac{1}{\theta} e^{-y/\theta}, \quad y > 0; \quad \theta > 0.$$

(a) Use moment-generating functions to show that the ratio $2n\overline{Y}/\theta$ has a chi square distribution with $2n$ df.
(b) Use the result in part (a) to derive a $100(1-\alpha)\%$ confidence interval for $\theta$.

**Question 7.5.16** When working properly, the amounts of cement that a filling machine puts into 25-kg bags have a stan- dard deviation ($\sigma$) of 1.0 kg. In the next column are the weights recorded for thirty bags selected at random from a day's production. Test $H_0 : \sigma^2 = 1$ versus $H_1 : \sigma^2 > 1$ using the $\alpha = 0.05$ level of significance. Assume that the weights are normally distributed.

| | | |
|---|---|---|
| 26.18 | 24.22 | 24.22 |
| 25.30 | 26.48 | 24.49 |
| 25.18 | 23.97 | 25.68 |
| 24.54 | 25.83 | 26.01 |
| 25.14 | 25.05 | 25.50 |
| 25.44 | 26.24 | 25.84 |
| 24.49 | 25.46 | 26.09 |
| 25.01 | 25.01 | 25.21 |
| 25.12 | 24.71 | 26.04 |
| 25.67 | 25.27 | 25.23 |

Use the following sums:

$$\sum_{i=1}^{30} y_i = 758.62 \quad \text{and} \quad \sum_{i=1}^{30} y_i^2 = 19,195.7938.$$