Probability and Statistics I

STAT 3600 – Fall 2021

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Chapter 2. Discrete Distributions

§ 2.1 Random Variables of the Discrete Type

§ 2.2 Mathematical Expectation

§ 2.3 Special Mathematical Expectation

§ 2.4 The Binomial Distribution

§ 2.5 The Hypergeometric Distribution

§ 2.6 The Negative Binomial Distribution

§ 2.7 The Poisson Distribution

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Definition 2.2-1 If $f(x)$ is the p.m.f. of the random variable X of the discrete type with space S, and if the summation

$$\sum_{x \in S} u(x) f(x), \quad \text{which is sometimes written} \quad \sum_{S} u(x) f(x),$$

exists, then the sum is called the *mathematical expectation* or the *expected value* of the function $u(X)$, and it is denoted by $\mathbb{E}[u(X)]$. That is,

$$\mathbb{E}[u(X)] = \sum_{x \in S} u(x) f(x).$$

Remark 2.2-1 The usual definition of mathematical expectation of $u(X)$ requires that the sum *converge absolutely*, that is, that

$$\sum_{x \in S} |u(x)| f(x)$$

converge and be finite. The reason for the absolute convergence is that it allows one, in the advanced proof of

$$\sum_{x \in S_X} |u(x)| f(x) = \sum_{y \in S_Y} |y| g(y),$$

to rearrange the order of the terms in the $x-$summation.

Example 2.2-1 Let the random variable X have the p.m.f.

$$f(x) = \frac{1}{5}, \quad x \in S,$$

where $S = \{-2, -1, 0, 1, 2\}$. Then find (a) $\mathbb{E}[X]$;

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where $S = \{-2, -1, 0, 1, 2\}$. Then find (c) $\mathbb{E}[X^3]$.

Theorem 2.2-1 When it exists, the mathematical expectation $\mathbb{E}$ satisfies the following properties:

(a) If c is a constant, then $\mathbb{E}(c) = c$.

(b) If c is a constant and u is a function, the

$$\mathbb{E}[cu(X)] = c\mathbb{E}[u(X)].$$

(c) If c_1 and c_2 are constants and u_1 and u_2 are functions, then

$$\mathbb{E}[c_1 u_1(X) + c_2 u_2(X)] = c_1\mathbb{E}[u(X)] + c_2\mathbb{E}[u_2(X)].$$

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Example 2.2-2 For the pmf given in Example 2.2-1, find (a) $\mathbb{E}[X(3-2X)]$;

Example 2.2-2 For the pmf given in Example 2.2-1, find (b) $\mathbb{E}[3X^2 + 4X^3 - 5]$.

Example 2.2-3 Let X have a *hypergeometric distribution* in which n objects are selected from $N = N_1 + N_2$, that is, the pmf is given by

$$f(x) = \mathbb{P}(X = x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}, \quad x \in \{0, 1, \cdots, n\}.$$

Then find $\mathbb{E}[X]$.

Example 2.2-4 In the casino game called *high-low*, there are three possible bets. Assume that \$1 is the size of the bet. A pair of fair six-sided dice is rolled and their sum is calculated. If you bet *low*, you win \$1 if the sum of the dice is $\{2, 3, 4, 5, 6\}$. If you bet *high*, you win \$1 if the sum of the dice is $\{8, 9, 10, 11, 12\}$. If you bet on $\{7\}$, you win \$4 if the sum is 7 is rolled. Otherwise, you lose on each of the three bets. In all three cases, your original dollar is returned if you win. Find the expected value of the game to the bettor for each of these three bets.

Exercises from textbook: 2.2-1, 2.2-2, 2.2-4-2.2-7, 2.2-11, 2.2-12