Probability and Statistics I

STAT 3600 – Fall 2021

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Chapter 2. Discrete Distributions

Definition 2.7-1 Let the number of occurrences of some event in a given continuous interval be counted. Then we have an *approximate Poisson process* with parameter $\lambda > 0$ if the following conditions are satisfied:

(a) The numbers of occurrences in non overlapping subintervals are independent.

(b) The probability of exactly one occurrence in a sufficiently short subinterval of length h is approximately λh.

(c) The probability of two or more occurrences in a sufficiently short subinterval is essentially zero.

Definition 2.7-2 A r.v. X is called a *Poisson* r.v. with parameter $\lambda(>0)$ if its pmf is given by

$$f(x) = \mathbb{P}(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}, \qquad x = 0, 1, 2, 3, \cdots .$$

The corresponding cdf of X is

$$F(x) = e^{-\lambda} \sum_{k=0}^{n} \frac{\lambda^k}{k!} \qquad n \leq x < n + 1.$$

The moment-generating function of Poisson r.v. X is

$$M(t) = \mathbb{E}(e^{tX}) = \sum_{x=0}^{\infty} e^{tx} \frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!} = e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)},$$

from which one obtain the mean and variance of the Poisson r.v. X

$$\mu = \mathbb{E}(X) = \lambda \quad \text{and} \quad \sigma^2 = \mathrm{Var}(X) = \lambda.$$

Remark 2.7-1 In the case of large n and small p, we have that

$$\binom{n}{k} p^k (1-p)^{n-k} \approx e^{-\lambda} \frac{\lambda^k}{k!} \qquad np = \lambda$$

which indicates that the binomial distribution can be approximated by the Poisson distribution.

The Poisson r.v. has a tremendous range of applications in diverse areas because it may be used as an approximation for binomial r.v. with parameters (n, p) when n is large and p is small enough so that np is of a moderate size.

Some examples of Poisson r.v.'s include

1. The number of telephone calls arriving at a switching center during various intervals of time
2. The number of misprints on a page of a book
3. The number of customers entering a bank during various intervals of time.

Example 2.7-1 A noisy transmission channel has a per-digit error probability $p = 0.01$.

(a) Calculate the probability of more than one error in 10 received digits.

(b) Repeat (a), using the Poisson approximation.

Example 2.7-2 The number of telephone calls arriving at a switchboard during any $10-$minute period is known to be a Poisson r.v. X with $\lambda = 2$.
(a) Find the probability that more than three calls will arrive during any 10-minute period.
(b) Find the probability that no calls will arrive during any $10-$minute period.

Exercises from textbook: 2.7-1, 2.7-2,2.7-3, 2.7-5,2.7-9, 2.7-11.