Probability and Statistics I

STAT 3600 – Fall 2021

Le Chen
lzc0090@auburn.edu

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Auburn University
Auburn AL

Chapter 4. Bivariate Distributions

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§ 4.1 Bivariate Distributions of the Discrete Type

§ 4.2 The Correlation Coefficient

§ 4.3 Conditional Distributions

§ 4.4 Bivariate Distributions of the Continuous Type

§ 4.5 The Bivariate Normal Distribution

Definition 4.3-1 Let X and Y have a joint discrete distribution with pmf $f(x, y)$ on space S. Say the marginal probability mass functions are $f_1(x)$, and $f_2(y)$ with space S_1 and S_2, respectively. The *conditional probability mass function of X*, given that $Y = y$, is defined by

$$g(x|y) = \frac{f(x, y)}{f_2(y)} \quad \text{provided that } f_2(y) > 0.$$

Similarly, the *conditional probability mass function of Y*, given that $X = x$, is defined by

$$h(y|x) = \frac{f(x, y)}{f_1(x)} \quad \text{provided that } f_1(x) > 0.$$

Definition 4.3-2 If (X, Y) is a discrete bivariate r.v. with joint pmf $f(x, y)$, then the *conditional mean (or conditional expectation)* of Y, given that $X = x$, is defined by

$$\mu_{Y|x} = \mathbb{E}(Y|x) = \sum_y yh(y|x),$$

and the *conditional variance* of Y, given that $X = x$, is defined by

$$\sigma^2_{Y|x} = \mathbb{E}\{[Y - \mathbb{E}(Y|x)]^2|x\} = \sum_y [y - \mathbb{E}(Y|x)]^2 h(y|x)$$

which can be reduced to

$$\text{Var}(Y|x) = \sigma^2_{Y|x} = \mathbb{E}(Y^2|x) - [E(Y|x)]^2 = \mathbb{E}(Y^2|x) - (\mu_{Y|x})^2.$$

Example 4.3-1 Let X and Y have a uniform distribution on the set of points with integer coordinates in $S = \{(x, y) : 0 \leq x \leq 7, x \leq y \leq x + 2\}$. That is, $f(x, y) = 1/24, (x, y) \in S$, and both x and y are integers.
(a) Find $f_1(x)$.

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(b) Find $h(y|x)$.

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(c) Find $\mu_{Y|x} = \mathbb{E}(Y|x)$.

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(d) Find $\sigma^2_{Y|x}$.

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(a) Find $f_1(x)$.
(b) Find $h(y|x)$.
(c) Find $\mu_{Y|x} = \mathbb{E}(Y|x)$.
(d) Find $\sigma^2_{Y|x}$.
Ans.
(a) $f_1(x) = 1/8, x = 0, 1, \cdots 7$;
(b) $h(y|x) = 1/3, y = x, x + 1, x + 2$, for $x = 0, 1, \cdots, 7$;
(c) $\mu_{Y|x} = x + 1, x = 0, 1, \cdots, 7$.
(d) $\sigma^2_{Y|x} = 2/3$.

Exercises from textbook: 4.3-1, 4.3-2, 4.3-5, 4.3-6, 4.3-10