

# Probability and Statistics I

STAT 3600 – Fall 2021

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Last updated on

July 4, 2021

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# Chapter 5. Distributions of Functions of Random Variables

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§ 5.1 Functions of One Random Variable

§ 5.2 Transformations of Two Random Variables

§ 5.3 Several Random Variables

§ 5.4 The Moment-Generating Function Technique

§ 5.5 Random Functions Associated with Normal Distributions

§ 5.6 The Central Limit Theorem

§ 5.7 Approximations for Discrete Distributions

§ 5.8 Chebyshev Inequality and Convergence in Probability

§ 5.9 Limiting Moment-Generating Functions

If  $X_1$  and  $X_2$  are two continuous-type random variables with joint pdf  $f(x_1, x_2)$ , and if

$$\begin{cases} Y_1 = u_1(X_1, X_2) \\ Y_2 = u_2(X_1, X_2) \end{cases}$$

has the single-valued inverse

$$\begin{cases} X_1 = v_1(Y_1, Y_2), \\ X_2 = v_2(Y_1, Y_2), \end{cases}$$

then the joint pdf of  $Y_1$  and  $Y_2$  is

$$g(y_1, y_2) = |J| f(v_1(y_1, y_2), v_2(y_1, y_2)), \quad (y_1, y_2) \in \mathcal{S}_1$$

where the **Jacobian**  $J$  is the determinant

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix}.$$

Example 5.2-1 Let  $X_1$  and  $X_2$  be independent random variables, each with pdf

$$f(x) = e^{-x}, \quad 0 < x < \infty.$$

Find the joint pdf of

$$\begin{cases} Y_1 = X_1 - X_2, \\ Y_2 = X_1 + X_2. \end{cases}$$

Find the pdf of  $Y_1$  and  $Y_2$ .

**Example 5.2-2** Let  $X$  and  $Y$  be independent uniform r.v.'s over  $(0, 1)$ . Find the pdf of  $Z = XY$ .