Probability and Statistics I

STAT 3600 – Fall 2021

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Last updated on

July 4, 2021

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Chapter 5. Distributions of Functions of Random Variables

If X_1 and X_2 are two continuous-type random variables with joint pdf $f(x_1, x_2)$, and if

$$\begin{cases} Y_1 = u_1(X_1, X_2) \\ Y_2 = u_2(X_1, X_2) \end{cases}$$

has the single-valued inverse

$$\begin{cases} X_1 = v_1(Y_1, Y_2), \\ X_2 = v_2(Y_1, Y_2), \end{cases}$$

then the joint pdf of Y_1 and Y_2 is

$$g(y_1, y_2) = |J| \, f\left(v_1(y_1, y_2), v_2(y_1, y_2)\right), \quad (y_1, y_2) \in S_1$$

where the Jacobian J is the determinant

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix}.$$

Example 5.2-1 Let X_1 and X_2 be independent random variables, each with pdf

$$f(x) = e^{-x}, \quad 0 < x < \infty.$$

Find the joint pdf of

$$\begin{cases} Y_1 = X_1 - X_2, \\ Y_2 = X_1 + X_2. \end{cases}$$

Find the pdf of Y_1 and Y_2.

Example 5.2-2 Let X and Y be independent uniform r.v.'s over $(0, 1)$. Find the pdf of $Z = XY$.