

Probability and Statistics I

STAT 3600 – Fall 2021

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Chapter 5. Distributions of Functions of Random Variables

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§ 5.1 Functions of One Random Variable

§ 5.2 Transformations of Two Random Variables

§ 5.3 Several Random Variables

§ 5.4 The Moment-Generating Function Technique

§ 5.5 Random Functions Associated with Normal Distributions

§ 5.6 The Central Limit Theorem

§ 5.7 Approximations for Discrete Distributions

§ 5.8 Chebyshev Inequality and Convergence in Probability

§ 5.9 Limiting Moment-Generating Functions

Theorem 5.4-1 If X_1, X_2, \dots, X_n are independent random variables with respective moment generating functions $M_{X_i}(t), i = 1, 2, 3, \dots, n$, then the moment-generating function of $Y = \sum_{i=1}^n a_i X_i$ is

$$M_Y(t) = \prod_{i=1}^n M_{X_i}(a_i t).$$

In particular, the moment-generating function of $\bar{X} = \sum_{i=1}^n (1/n) X_i$ is

$$M_{\bar{X}}(t) = \prod_{i=1}^n M_{X_i}(t/n).$$

Example 5.4-1 Let X_1 and X_2 have independent distributions $b(n_1, p)$ and $b(n_2, p)$. Find the moment-generating function of $Y = X_1 + X_2$. How is Y distributed?

Example 5.4-2 Let X_1, X_2, X_3, X_4, X_5 be a random sample of size 5 from a geometric distribution with $p = 1/3$.

(a) Find the moment generating function of $Y = X_1 + X_2 + X_3 + X_4 + X_5$.

(b) How is Y distributed?

(c) Find mgf of \bar{X} .

Ans: (a) ... (b) ... (c) ...

Theorem 5.4-2 Let X_1, X_2, \dots, X_n be independent chi-square random variables with r_1, r_2, \dots, r_n degrees of freedom, respectively. Then $Y = X_1 + X_2 + \dots + X_n$ follows $\chi^2(r_1 + r_2 + \dots + r_n)$.

Corollary 5.4-3 Let Z_1, Z_2, \dots, Z_n be independent standard normal distributions, $N(0, 1)$, random variables, then $W = Z_1^2 + Z_2^2 + \dots + Z_n^2 \sim \chi^2(n)$.

Corollary 5.4-4 If X_1, X_2, \dots, X_n be independent and have a normal distributions $N(\mu_i, \sigma_i^2)$, $i = 1, 2, \dots, n$, respectively, then

$$W = \sum_{i=1}^n \frac{(X_i - \mu_i)^2}{\sigma_i^2} \sim \chi^2(n).$$

Exercises from textbook: 5.4.1, 5.4-3, 5.4.4, 5.4-7, 5.4-8, 5.4-15.