# Probability and Statistics I

STAT 3600 – Fall 2021

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Last updated on

July 4, 2021

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# Chapter 5. Distributions of Functions of Random Variables

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Theorem 5.4-1 If $X_1, X_2, \cdots X_n$ are independent random variables with respective moment generating functions $M_{X_i}(t), i = 1, 2, 3, \cdots, n$, then the moment-generating function of $Y = \sum_{i=1}^{n} a_i X_i$ is

$$M_Y(t) = \prod_{i=1}^{n} M_{X_i}(a_i t).$$

In particular, the moment-generating function of $\overline{X} = \sum_{i=1}^{n} (1/n) X_i$ is

$$M_{\overline{X}}(t) = \prod_{i=1}^{n} M_{X_i}(t/n).$$

Example 5.4-1 Let $X_1$ and $X_2$ have independent distributions $b(n_1, p)$ and $b(n_2, p)$. Find the moment-generating function of $Y = X_1 + X_2$. How is $Y$ distributed?

Example 5.4-2 Let $X_1, X_2, X_3, X_4, X_5$ be a random sample of size 5 from a geometric distribution with $p = 1/3$.

(a) Find the moment generating function of $Y = X_1 + X_2 + X_3 + X_4 + X_5$.

(b) How is $Y$ distributed?

(c) Find mgf of $\overline{X}$.

Ans: (a) ... (b) ... (c) ...

Theorem 5.4-2 Let $X_1, X_2, \cdots, X_n$ be independent chi-square random variables with $r_1, r_2, \cdots, r_n$ degrees of freedom, respectively. Then $Y = X_1 + X_2 + \cdots + X_n$ follows $\chi^2(r_1 + r_2 + \cdots + r_n)$.

**Corollary 5.4-3** Let $Z_1, Z_2, \cdots, Z_n$ be independent standard normal distributions, $N(0,1)$, random variables, then $W = Z_1^2 + Z_2^2 + \cdots + Z_n^2 \sim \chi^2(n)$.

**Corollary 5.4-4** If $X_1, X_2, \cdots, X_n$ be independent and have a normal distributions $N(\mu_i, \sigma_i^2), i = 1, 2, \cdots, n$, respectively, then

$$W = \sum_{i=1}^{n} \frac{(X_i - \mu_i)^2}{\sigma_i^2} \sim \chi^2(n).$$

Exercises from textbook: 5.4.1, 5.4-3, 5.4.4, 5.4-7, 5.4-8, 5.4-15.