

Probability and Statistics I

STAT 3600 – Fall 2021

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Chapter 5. Distributions of Functions of Random Variables

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§ 5.1 Functions of One Random Variable

§ 5.2 Transformations of Two Random Variables

§ 5.3 Several Random Variables

§ 5.4 The Moment-Generating Function Technique

§ 5.5 Random Functions Associated with Normal Distributions

§ 5.6 The Central Limit Theorem

§ 5.7 Approximations for Discrete Distributions

§ 5.8 Chebyshev Inequality and Convergence in Probability

§ 5.9 Limiting Moment-Generating Functions

Theorem 5.6-1 (Central Limit Theorem) If \bar{X} is the mean of a random sample X_1, X_2, \dots, X_n of size n from a distribution with a finite mean μ and a finite positive variance σ^2 , then the distribution of

$$W = \frac{\bar{X} - \mu}{\sqrt{\sigma/n}} = \frac{\sum_{i=1}^n X_i - n\mu}{\sqrt{n}\sigma}$$

is $N(0, 1)$ in the limit as $n \rightarrow \infty$.

When n is "sufficiently large," a practical use of the central limit theorem is approximating the cdf of W , namely,

$$\mathbb{P}(W \leq w) \approx \int_{-\infty}^w \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = \Phi(w).$$

Example 5.6-1 Let \bar{X} be the mean of a random sample of size 12 from the uniform distribution on the interval $(0, 1)$. Approximate $\mathbb{P}(1/2 \leq \bar{X} \leq 2/3)$.

Example 5.6-2 Let X equal the weight in grams of a miniature candy bar. Assume that $\mu = \mathbb{E}(X) = 24.43$ and $\sigma^2 = \text{Var}(X) = 2.20$. Let \bar{X} be the sample mean of a random of $n = 30$ candy bars.

(a) Find $\mathbb{E}(\bar{X})$;

(b) Find $\text{Var}(\bar{X})$;

(c) Find $\mathbb{P}(24.17 \leq \bar{X} \leq 24.82)$ approximately.

Ans: (a) ... (b) ... (c) ...

Exercises from textbook: 5.6-2, 5.6-4, 5.6-6, 5.6-7, 5.6-9.