Probability and Statistics I

STAT 3600 – Fall 2021

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Chapter 5. Distributions of Functions of Random Variables

Theorem 5.6-1 (Central Limit Theorem) If $\overline{X}$ is the mean of a random sample $X_1, X_2, \cdots, X_n$ of size n from a distribution with a finite mean μ and a finite positive variance σ^2, then the distribution of

$$W = \frac{\overline{X} - \mu}{\sqrt{\sigma}/n} = \frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n}\sigma}$$

is $N(0, 1)$ in the limit as $n \to \infty$.

When n is "sufficiently large," a practical use of the central limit theorem is approximating the cdf of W, namely,

$$\mathbb{P}(W \leq w) \approx \int_{-\infty}^{w} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = \Phi(w).$$

Example 5.6-1 Let $\overline{X}$ be the mean of a random sample of size 12 from the uniform distribution on the interval $(0, 1)$. Approximate $\mathbb{P}(1/2 \leq \overline{X} \leq 2/3)$.

Example 5.6-2 Let X equal the weight in grams of a miniature candy bar. Assume that $\mu = \mathbb{E}(X) = 24.43$ and $\sigma^2 = Var(X) = 2.20$. Let $\overline{X}$ be the sample mean of a random of $n = 30$ candy bars.

(a) Find $\mathbb{E}\left(\overline{X}\right)$;

(b) Find $\mathrm{Var}\left(\overline{X}\right)$;

(c) Find $\mathbb{P}\left(24.17 \leq \overline{X} \leq 24.82\right)$ approximately.

Ans: (a) ... (b) ... (c) ...

Exercises from textbook: 5.6-2, 5.6-4, 5.6-6, 5.6-7, 5.6-9.