Exercises for 5.7

Exercise 5.7.1 The following table gives IQ scores for 10 fathers and their eldest sons. Calculate the means, the variances, and the correlation coefficient r. (The data scaling formula is useful.)

	1	2	3	4	5	6	7	8	9	10
Father's IQ	140	131	120	115	110	106	100	95	91	86
Son's IQ	130	138	110	99	109	120	105	99	100	94

Exercise 5.7.2 The following table gives the number of years of education and the annual income (in thousands) of 10 individuals. Find the means, the variances, and the correlation coefficient. (Again the data scaling formula is useful.)

Individual	1	2	3	4	5	6	7	8	9	10
Years of education	12	16	13	18	19	12	18	19	12	14
Yearly income (1000's)	31	48	35	28	55	40	39	60	32	35

Exercise 5.7.3 If $\mathbf{x}$ is a sample vector, and $\mathbf{x}_c$ is the centred sample, show that $\overline{x}_c = 0$ and the standard deviation of $\mathbf{x}_c$ is s_x.

Exercise 5.7.4 Prove the data scaling formulas found on page 326: (a), (b), and (c).

Supplementary Exercises for Chapter 5

Exercise 5.1 In each case either show that the statement is true or give an example showing that it is false. Throughout, $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, $\mathbf{x}_1$, $\mathbf{x}_2$, $\dots$, $\mathbf{x}_n$ denote vectors in $\mathbb{R}^n$.

a. If U is a subspace of $\mathbb{R}^n$ and $\mathbf{x}+\mathbf{y}$ is in U, then $\mathbf{x}$ and $\mathbf{y}$ are both in U.

b. If U is a subspace of $\mathbb{R}^n$ and $r\mathbf{x}$ is in U, then $\mathbf{x}$ is in U.

c. If U is a nonempty set and $s\mathbf{x}+t\mathbf{y}$ is in U for any s and t whenever $\mathbf{x}$ and $\mathbf{y}$ are in U, then U is a subspace.

d. If U is a subspace of $\mathbb{R}^n$ and $\mathbf{x}$ is in U, then $-\mathbf{x}$ is in U.

e. If $\{\mathbf{x}, \mathbf{y}\}$ is independent, then $\{\mathbf{x}, \mathbf{y}, \mathbf{x}+\mathbf{y}\}$ is independent.

f. If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent, then $\{\mathbf{x}, \mathbf{y}\}$ is independent.

g. If $\{\mathbf{x}, \mathbf{y}\}$ is not independent, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is not independent.

h. If all of $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ are nonzero, then $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is independent.

i. If one of $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ is zero, then $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is not independent.

j. If $a\mathbf{x}+b\mathbf{y}+c\mathbf{z}=\mathbf{0}$ where a, b, and c are in $\mathbb{R}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent.

k. If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is independent, then $a\mathbf{x}+b\mathbf{y}+c\mathbf{z}=\mathbf{0}$ for some a, b, and c in $\mathbb{R}$.

l. If $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is not independent, then $t_1\mathbf{x}_1+t_2\mathbf{x}_2+\cdots+t_n\mathbf{x}_n=\mathbf{0}$ for t_i in $\mathbb{R}$ not all zero.

m. If $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ is independent, then $t_1\mathbf{x}_1+t_2\mathbf{x}_2+\cdots+t_n\mathbf{x}_n=\mathbf{0}$ for some t_i in $\mathbb{R}$.

n. Every set of four non-zero vectors in $\mathbb{R}^4$ is a basis.

o. No basis of $\mathbb{R}^3$ can contain a vector with a component $\mathbf{0}$.

p. $\mathbb{R}^3$ has a basis of the form $\{\mathbf{x}, \mathbf{x}+\mathbf{y}, \mathbf{y}\}$ where $\mathbf{x}$ and $\mathbf{y}$ are vectors.

q. Every basis of $\mathbb{R}^5$ contains one column of I_5.

r. Every nonempty subset of a basis of $\mathbb{R}^3$ is again a basis of $\mathbb{R}^3$.

s. If $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4\}$ and $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3, \mathbf{y}_4\}$ are bases of $\mathbb{R}^4$, then $\{\mathbf{x}_1+\mathbf{y}_1, \mathbf{x}_2+\mathbf{y}_2, \mathbf{x}_3+\mathbf{y}_3, \mathbf{x}_4+\mathbf{y}_4\}$ is also a basis of $\mathbb{R}^4$.