Math 221: LINEAR ALGEBRA

Chapter 1. Systems of Linear Equations

§1-6. Application to Chemical Reactions

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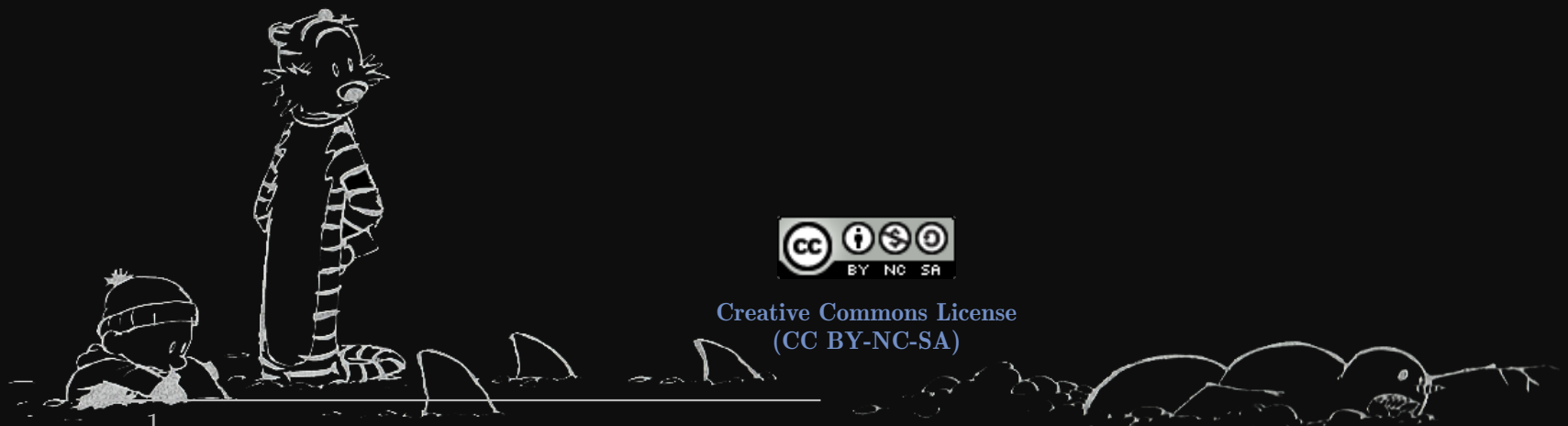

[1] Slides are adapted from those by Karen Seyffarth from University of Calgary.

Chemical Reactions

Balancing Chemical Reactions

Problem

Balance the chemical reaction given below involving tin (Sn), hydrogen (H), and oxygen (0).

$$\mathrm{x}SnO_2 + \mathrm{y}H_2 \rightarrow \mathrm{z}Sn + \mathrm{w}H_2O$$

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Solution

Setting up a system of equations in $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{w}$ gives

$$\begin{aligned} \mathrm{Sn} &: \quad \mathrm{x} = \mathrm{z} \text{ or } \mathrm{x} - \mathrm{z} = 0 \\ \mathrm{O} &: \quad 2\mathrm{x} = \mathrm{w} \text{ or } 2\mathrm{x} - \mathrm{w} = 0 \\ \mathrm{H} &: \quad 2\mathrm{y} = 2\mathrm{w} \text{ or } 2\mathrm{y} - 2\mathrm{w} = 0 \end{aligned}$$

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The augmented matrix is $\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 \end{array}\right]$

Solution (continued)

The reduced row-echelon matrix is

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array}\right]$$

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Letting $w = t$, the solution is

$$\begin{aligned} x &= \frac{1}{2}t \\ y &= t \\ z &= \frac{1}{2}t \\ w &= t \end{aligned}$$

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We can choose any values for $w = t$. Suppose we choose $w = 4$, then $x = 2, y = 4, z = 2$ and the balanced reaction is

$$2Sn0_2 + 4H_2 \rightarrow 2Sn + 4H_2O$$