Math 221: LINEAR ALGEBRA

Chapter 2. Matrix Algebra
§2-1. Matrix Addition, Scalar Multiplication and Transposition

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Emory University, 2021 Spring

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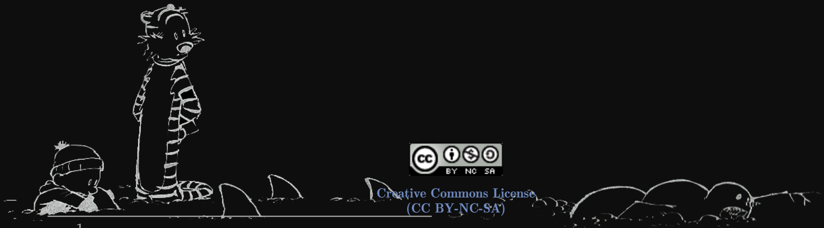

[1] Slides are adapted from those by Karen Seyffarth from University of Calgary.

Matrices – Definitions and Basic Properties

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General notation for an $m \times n$ matrix, A:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} = [a_{ij}]$$

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6. Subtraction: for $m \times n$ matrices A and B, $A - B = A + (-1)B$.

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Scalar Multiplication

The Transpose

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Example

Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 \\ 6 & 1 \end{bmatrix}$. Then,

$$\begin{aligned} A + B &= \begin{bmatrix} 1+0 & 3+-2 \\ 2+6 & 5+1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 8 & 6 \end{bmatrix} \end{aligned}$$

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4. There exists an $m \times n$ matrix $-A$ such that $A + (-A) = 0$. (existence of an additive inverse).

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Then

$$\begin{aligned} 3A &= \begin{bmatrix} 3(2) & 3(0) & 3(-1) \\ 3(3) & 3(1) & 3(-2) \\ 3(0) & 3(4) & 3(5) \end{bmatrix} \\ &= \begin{bmatrix} 6 & 0 & -3 \\ 9 & 3 & -6 \\ 0 & 12 & 15 \end{bmatrix} \end{aligned}$$

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4. $1A = A$. (existence of a multiplicative identity).

Example

$$2\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} + 4\begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 6 & 8 \\ 1 & -1 \end{bmatrix} =$$

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Problem

Let A and B be $m \times n$ matrices. Simplify the expression

$$2[9(A - B) + 7(2B - A)] - 2[3(2B + A) - 2(A + 3B) - 5(A + B)]$$

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Solution

$$\begin{aligned} & 2[9(A - B) + 7(2B - A)] - 2[3(2B + A) - 2(A + 3B) - 5(A + B)] \\ = \; & 2(9A - 9B + 14B - 7A) - 2(6B + 3A - 2A - 6B - 5A - 5B) \\ = \; & 2(2A + 5B) - 2(-4A - 5B) \\ = \; & 12A + 20B \end{aligned}$$

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Examples

$$\begin{bmatrix} 2 & -3 \\ -3 & 17 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 5 \\ 0 & 2 & 11 \\ 5 & 11 & -3 \end{bmatrix}, \begin{bmatrix} 0 & 2 & 5 & -1 \\ 2 & 1 & -3 & 0 \\ 5 & -3 & 2 & -7 \\ -1 & 0 & -7 & 4 \end{bmatrix}$$

are symmetric matrices, and each is symmetric about its main diagonal.

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Solution

We must show that $(A - A^T)^T = -(A - A^T)$. Using the properties of matrix addition, scalar multiplication, and transposition

$$(A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T).$$