

Math 362: Mathematical Statistics II

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Last updated on April 13, 2021

2021 Spring

Chapter 13. Randomized Block Designs

§ 13.1 Introduction

§ 13.2 The F Test for a Randomized Block Design

§ 13.A Appendix: Some Discussions and Extensions

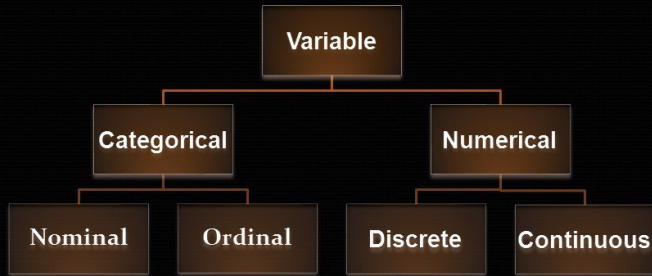
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Types of Variable with respect to data



Gender

1. Male
2. Female

Motivation

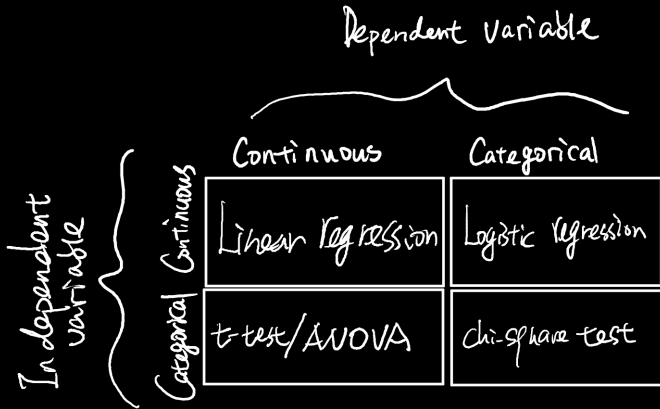
1. Highly Motivated
2. Moderately Motivated
3. Less Motivated

1. No of students
2. No of chairs
3. Collar size

1. Height
2. Weight
3. speed

	Numerical Values	Categorical Values	
n Samples	Sample 1	$y_{11} \dots y_{1m}$	$c_{11} \dots c_{1p}$
	Sample 2	$y_{21} \dots y_{2m}$	$c_{21} \dots c_{2p}$
	\vdots	\vdots	\vdots
	Sample n	$y_{n1} \dots y_{nm}$	$c_{n1} \dots c_{np}$

m
 p



Indep. v.s. Dependent

1. Categorical v.s. Continuous

1.1 $p = 1, m = 1,$	One-way ANOVA
1.2 $p = 2, m = 1,$	Two-way ANOVA
1.3 $p \geq 3, m = 1,$	p -way ANOVA
1.4 $p = 1, m \geq 2,$	One-way MANOVA ^a
1.5 $p = 2, m \geq 2,$	Two-way MANOVA
1.6 $p \geq 3, m \geq 2,$	p -way ANOVA

^aMANOVA refers to the multivariate analysis of variance
ANOVA refers to the univariate analysis of variance.

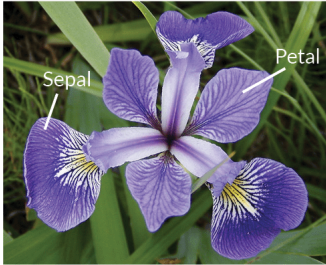
2. Continuous v.s. Continuous

2.1 $m_{ind} = 1, m_{dep} = 1,$	Simple linear regression
2.2 $m_{ind} \geq 2$	Multiple linear regression
2.3 $m_{dep} \geq 2$	Multivariate linear regression

E.g. One example for MANOVA¹.



¹<http://www.sthda.com/english/wiki/manova-test-in-r-multivariate-analysis-of-variance>



Iris Versicolor



Iris Setosa



Iris Virginica

```

1 > library(datasets)
2 > data(iris)
3 > summary(iris)
4   Sepal.Length Sepal.Width Petal.Length Petal.Width
   Species
5   Min.   :4.300   Min.   :2.000   Min.   :1.000   Min.   :0.100   setosa
   :50
6   1st Qu.:5.100   1st Qu.:2.800   1st Qu.:1.600   1st Qu.:0.300   versicolor
   :50
7   Median :5.800   Median :3.000   Median :4.350   Median :1.300   virginica
   :50
8   Mean   :5.843   Mean   :3.057   Mean   :3.758   Mean   :1.199
9   3rd Qu.:6.400   3rd Qu.:3.300   3rd Qu.:5.100   3rd Qu.:1.800
10  Max.   :7.900   Max.   :4.400   Max.   :6.900   Max.   :2.500
11 > my_data <- iris
12 > my_data
13   Sepal.Length Sepal.Width Petal.Length Petal.Width Species
14 1           5.1           3.5           1.4           0.2     setosa
15 2           4.9           3.0           1.4           0.2     setosa
16 3           4.7           3.2           1.3           0.2     setosa
17 4           4.6           3.1           1.5           0.2     setosa
18 5           5.0           3.6           1.4           0.2     setosa
19 6           5.4           3.9           1.7           0.4     setosa
20 7           4.6           3.4           1.4           0.3     setosa
21 8           5.0           3.4           1.5           0.2     setosa
22 9           4.4           2.9           1.4           0.2     setosa
23 10          4.9           3.1           1.5           0.1     setosa

```

```

1 > # Compute MAOVA test now
2 > res.man <- manova(cbind(Sepal.Length, Petal.Length) ~ Species, data =
  iris)
3 > summary(res.man)
4           Df Pillai approx F num Df den Df Pr(>F)
5 Species      2 0.9885  71.829      4   294 < 2.2e-16 ***
6 Residuals 147
7 ----
8 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
9 > # Look to see which differ
10 > summary.aov(res.man)
11   Response Sepal.Length :
12           Df Sum Sq Mean Sq F value Pr(>F)
13 Species      2 63.212 31.606 119.26 < 2.2e-16 ***
14 Residuals 147 38.956  0.265
15 ----
16 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
17
18   Response Petal.Length :
19           Df Sum Sq Mean Sq F value Pr(>F)
20 Species      2 437.10 218.551 1180.2 < 2.2e-16 ***
21 Residuals 147 27.22  0.185
22 ----
23 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1:w

```

Concl.: Two variables are highly significantly different among species.