Math 362: Mathematical Statistics II

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Chapter 13. Randomized Block Designs

Plan

§ 13.1 Introduction

§ 13.2 The F Test for a Randomized Block Design

§ 13.A Appendix: Some Discussions and Extensions

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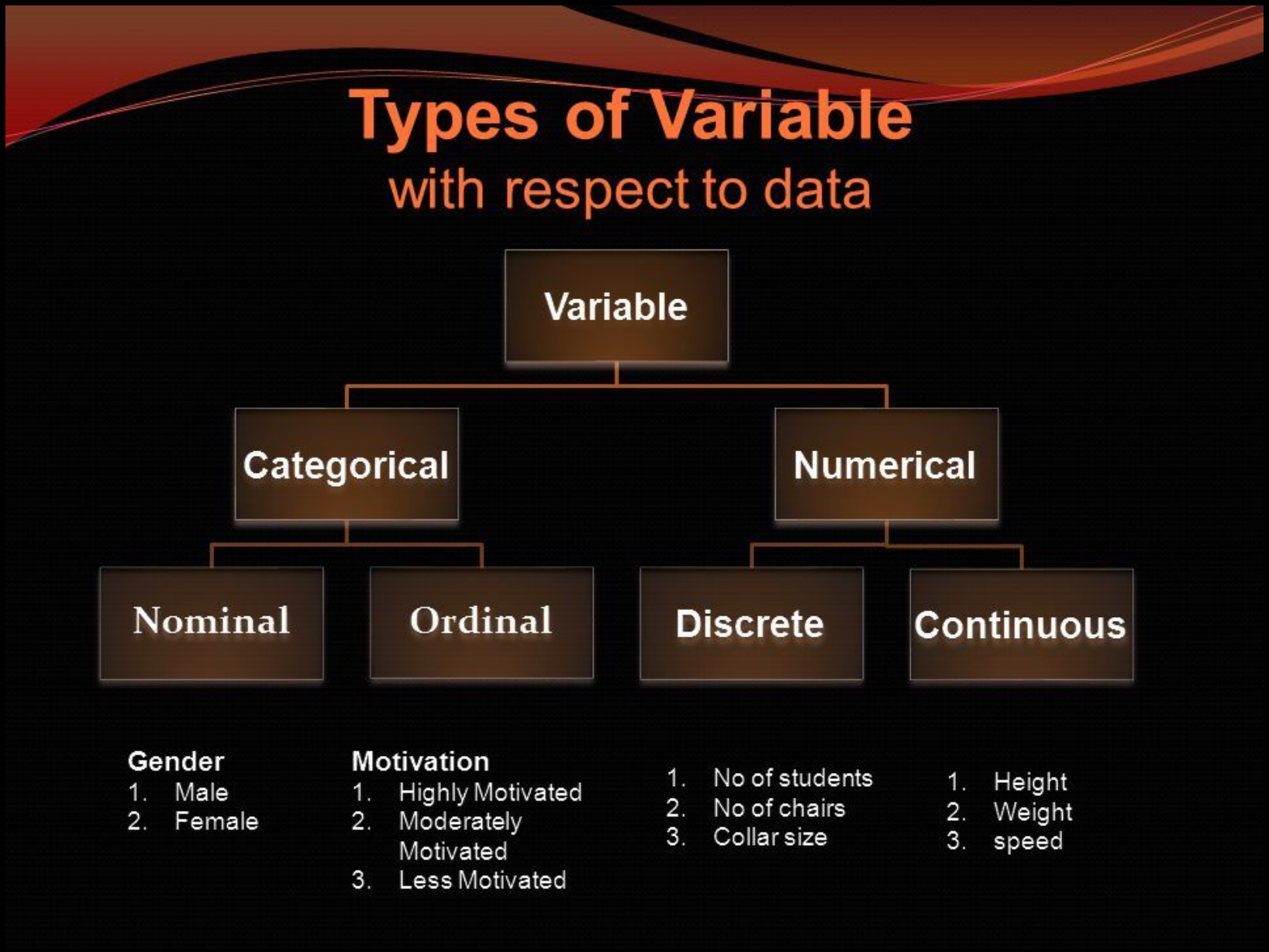
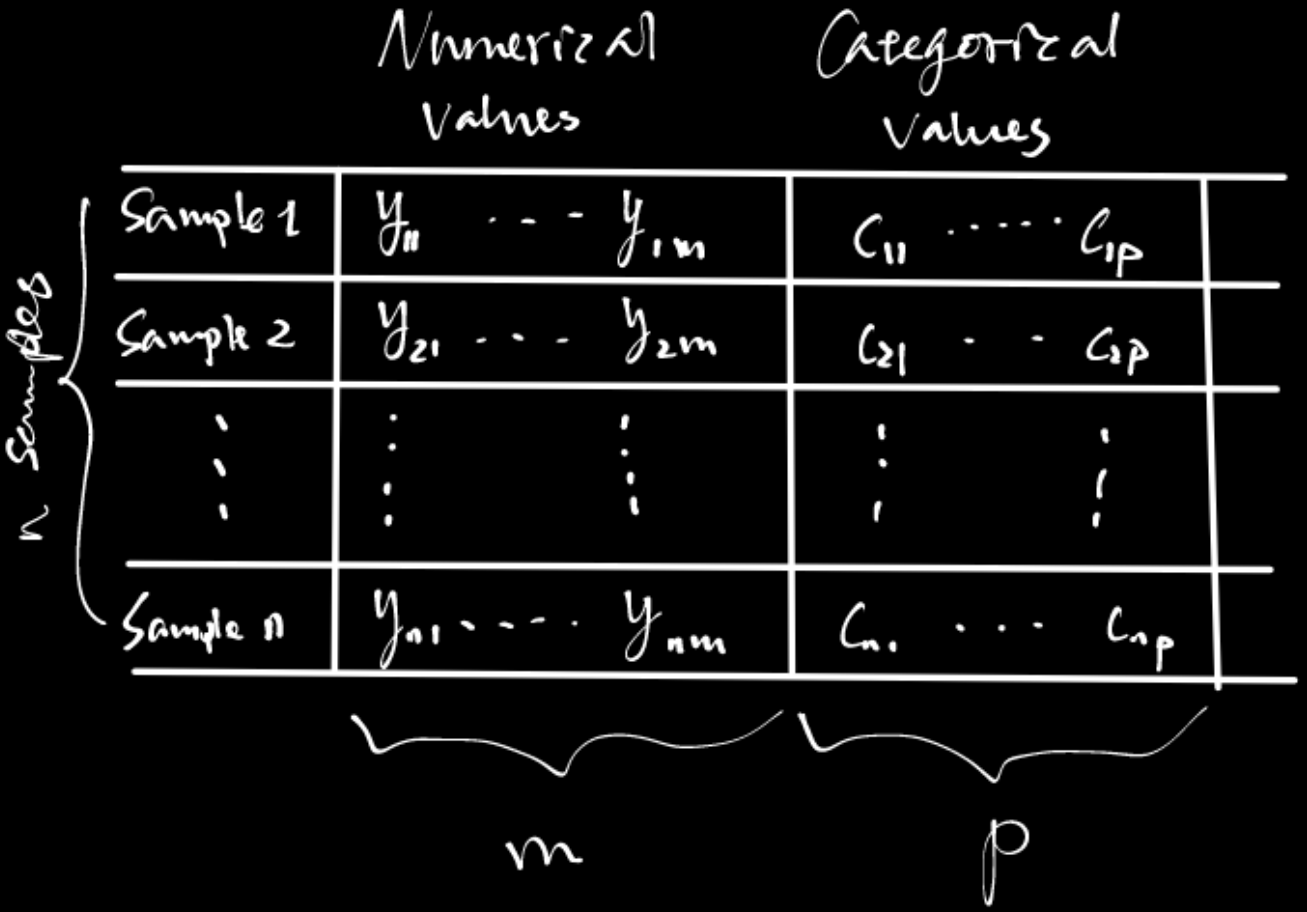
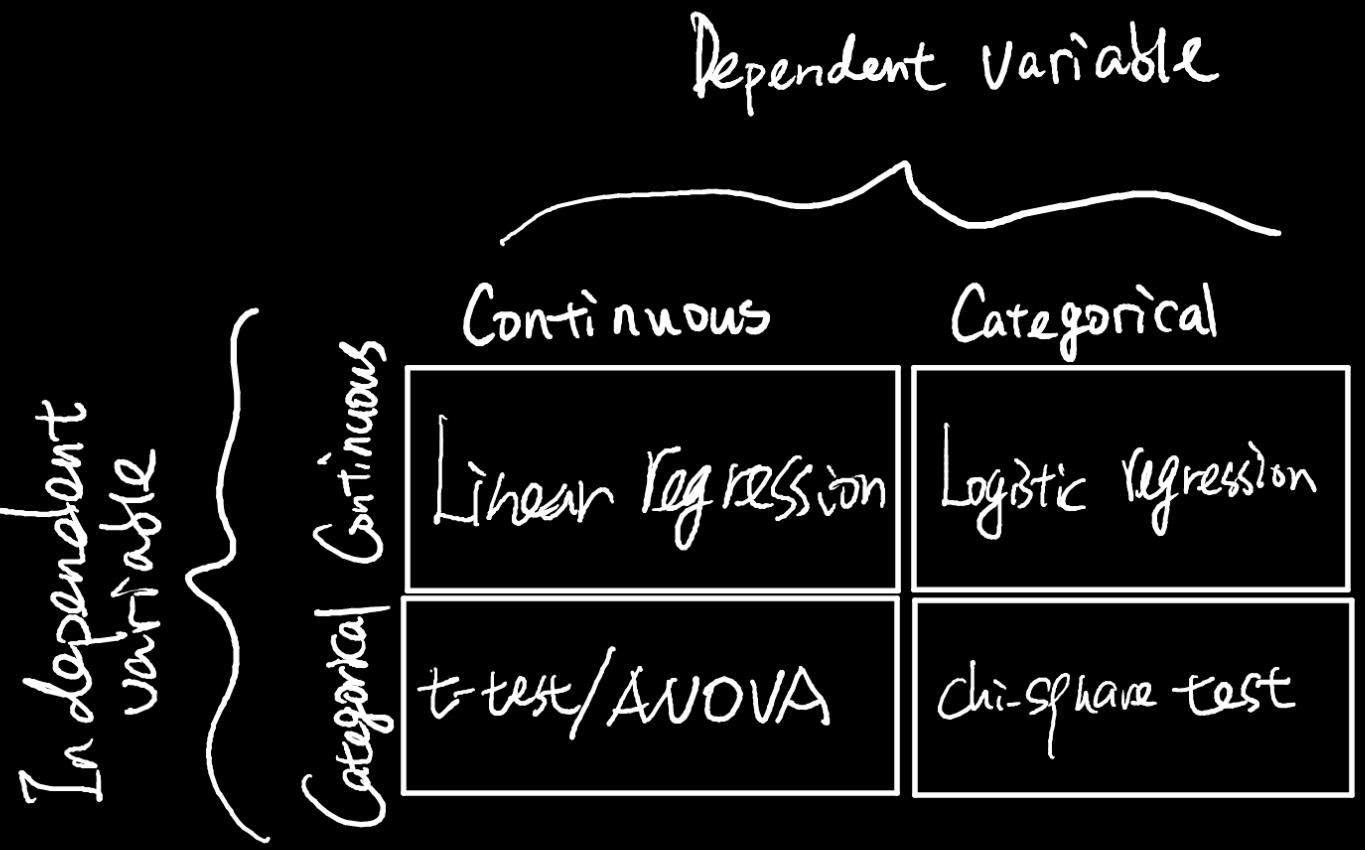



Numerical Values
Categorical Values
n Samples
Sample 1
Sample 2
Sample n
y_{11} ... y_{1m}
y_{21} ... y_{2m}
y_{n1} ... y_{nm}
c_{11} ... c_{1p}
c_{21} ... c_{2p}
c_{n1} ... c_{np}
m
p


Dependent variable
Continuous
Categorical
Independent variable
Categorical
Continuous
Linear regression
Logistic regression
t-test/ANOVA
Chi-square test

Indep. v.s. Dependent

1. Categorical v.s. Continuous

   1.1 $p = 1$, $m = 1$. — One-way ANOVA

   1.2 $p = 2$, $m = 1$. — Two-way ANOVA

   1.3 $p \geq 3$, $m = 1$. — p-way ANOVA

   1.4 $p = 1$, $m \geq 2$. — One-way MANOVA[2]

   1.5 $p = 2$, $m \geq 2$. — Two-way MANOVA

   1.6 $p \geq 3$, $m \geq 2$. — p-way ANOVA

2. Continuous v.s. Continuous

   2.1 $m_{\text{dep}} = 1$, $m_{\text{ind}} = 1$. — Simple linear regression

   2.2 $m_{\text{dep}} = 2$ — Multiple linear regression

   2.3 $m_{\text{dep}} = 2$ — Multivariate multiple regression[2]

[2] MANOVA refers to the multivariate analysis of variance
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E.g. One example for MANOVA[1].

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The Iris Dataset

Collected by Ronald Fisher in 1936

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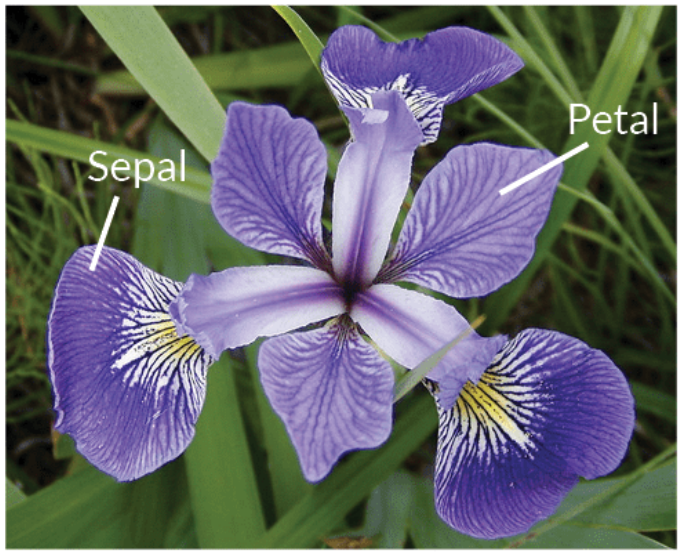

Iris Versicolor

Iris Setosa

Iris Virginica

```
> library(datasets)
> data(iris)
> summary(iris)
  Sepal.Length  Sepal.Width   Petal.Length  Petal.Width
        Species
 Min.  :4.300   Min.  :2.000   Min.  :1.000   Min.  :0.100   setosa
       :50
 1st Qu.:5.100  1st Qu.:2.800  1st Qu.:1.600  1st Qu.:0.300  versicolor
       :50
 Median :5.800 Median :3.000 Median :4.350 Median :1.300 virginica
       :50
 Mean :5.843   Mean :3.057   Mean :3.758   Mean :1.199
 3rd Qu.:6.400 3rd Qu.:3.300 3rd Qu.:5.100 3rd Qu.:1.800
 Max. :7.900   Max. :4.400   Max. :6.900   Max. :2.500
> my_data <- iris
> my_data
    Sepal.Length Sepal.Width Petal.Length Petal.Width Species
1            5.1         3.5          1.4         0.2    setosa
2            4.9         3.0          1.4         0.2    setosa
3            4.7         3.2          1.3         0.2    setosa
4            4.6         3.1          1.5         0.2    setosa
5            5.0         3.6          1.4         0.2    setosa
6            5.4         3.9          1.7         0.4    setosa
7            4.6         3.4          1.4         0.3    setosa
8            5.0         3.4          1.5         0.2    setosa
9            4.4         2.9          1.4         0.2    setosa
10           4.9         3.1          1.5         0.1    setosa
```

```
> # Compute MAOVA test now
> res.man <- manova(cbind(Sepal.Length, Petal.Length) ~ Species, data =
      iris)
> summary(res.man)
           Df Pillai approx F num Df den Df Pr(>F)
Species     2 0.9885   71.829      4    294 < 2.2e-16 ***
Residuals 147
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> # Look to see which differ
> summary.aov(res.man)
 Response Sepal.Length :
              Df Sum Sq Mean Sq F value Pr(>F)
Species       2 63.212  31.606  119.26 < 2.2e-16 ***
Residuals   147 38.956   0.265
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Response Petal.Length :
              Df Sum Sq Mean Sq F value Pr(>F)
Species       2 437.10 218.551  1180.2 < 2.2e-16 ***
Residuals   147  27.22   0.185
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1:w
```

Concl.: Two variables are highly significantly different among species.