

Math 362: Mathematical Statistics II

Le Chen

le.chen@emory.edu

Emory University
Atlanta, GA

Last updated on April 13, 2021

2021 Spring

Chapter 5. Estimation

§ 5.1 Introduction

§ 5.2 Estimating parameters: MLE and MME

§ 5.3 Interval Estimation

§ 5.4 Properties of Estimators

§ 5.5 Minimum-Variance Estimators: The Cramér-Rao Lower Bound

§ 5.6 Sufficient Estimators

§ 5.7 Consistency

§ 5.8 Bayesian Estimation

Plan

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Motivating example: Given an unfair coin, or p -coin, such that

$$X = \begin{cases} 1 & \text{head with probability } p, \\ 0 & \text{tail with probability } 1 - p, \end{cases}$$

how would you determine the value p ?

Solutions:

1. You need to try the coin several times, say, three times. What you obtain is “HHT”.
2. Draw a conclusion from the experiment you just made.

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Rationale: The choice of the parameter ρ should be the value that maximizes the probability of the sample.

$$\begin{aligned}\mathbb{P}(X_1 = 1, X_2 = 1, X_3 = 0) &= P(X_1 = 1)P(X_2 = 1)P(X_3 = 0) \\ &= \rho^2(1 - \rho).\end{aligned}$$

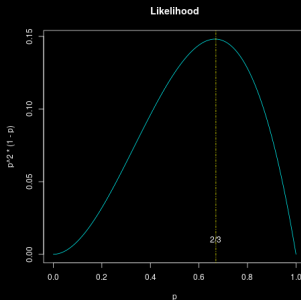
```
1 # Hello, R
2 p <- seq(0,1,0.01)
3 plot(p,p^2*(1-p),
4      type="l",
5      col="red")
6 title("Likelihood")
7 # add a vertical dotted (1) blue
   line
8 abline(v=0.67, col="blue", lty=4)
9 # add some text
10 text(0.67,0.01, "2/3")
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Maximize $f(\rho) = \rho^2(1 - \rho) \dots$

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Maximize $f(p) = p^2(1 - p) \dots$

A random sample of size n from the population – Bernoulli(p):

- ▶ X_1, \dots, X_n are i.i.d.¹ random variables, each following Bernoulli(p).
- ▶ Suppose the outcomes of the random sample are: $X_1 = k_1, \dots, X_n = k_n$.
- ▶ What is your choice of p based on the above random sample?

$$\hat{p} = \frac{1}{n} \sum_{i=1}^n k_i =: \bar{k}.$$

¹independent and identically distributed

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A random sample of size n from the population with given pdf:

- ▶ X_1, \dots, X_n are i.i.d. random variables, each following the same given pdf.
- ▶ a **statistic** or an **estimator** is a function of the random sample.

Statistic/Estimator is a random variable!

e.g.,

$$\hat{p} = \frac{1}{n} \sum_{i=1}^n X_i.$$

- ▶ The outcome of a statistic/estimator is called an **estimate**. e.g.,

$$p_e = \frac{1}{n} \sum_{i=1}^n k_i.$$

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