Math 362: Mathematical Statistics II

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Chapter 9. Two-Sample Inferences

Plan

Similar to the hypothesis test ...

1. Let $X_1, \cdots, X_n$ be a random sample of size n from $N(\mu_X, \sigma_X^2)$.

2. Let $Y_1, \cdots, Y_m$ be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

Prob. 1 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2 / σ_Y^2 or σ_X^2, σ_Y^2

1. Let $X_1, \cdots, X_n$ be a random sample of size n from $N(\mu_X, \sigma_X^2)$.

2. Let $Y_1, \cdots, Y_m$ be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

Prob. 1 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$

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When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2 or σ_X/σ_Y

1. Let $X_1, \cdots, X_n$ be a random sample of size n from $N(\mu_X, \sigma_X^2)$.

2. Let $Y_1, \cdots, Y_m$ be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

Prob. 1 Find the $100(1 - \alpha)\%$ C.I. for $\mu_X - \mu_Y$

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1 - \alpha)\%$ C.I. for σ_X^2 / σ_Y^2 or σ_X^2, σ_Y^2

1. Let $X_1, \cdots, X_n$ be a random sample of size n from $N(\mu_X, \sigma_X^2)$.

2. Let $Y_1, \cdots, Y_m$ be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

Prob. 1 Find the $100(1 - \alpha)\%$ C.I. for $\mu_X - \mu_Y$

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the 100(1 − α)% C.I. for σ²_X / σ²_Y or σ_X, σ_Y

1. Let $X_1, \cdots, X_n$ be a random sample of size n from $N(\mu_X, \sigma_X^2)$.

2. Let $Y_1, \cdots, Y_m$ be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

Prob. 1 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2 or σ_X, σ_Y

1. Let $X_1, \cdots, X_n$ be a random sample of size n from $N(\mu_X, \sigma_X^2)$.

2. Let $Y_1, \cdots, Y_m$ be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

Prob. 1 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for $\sigma_X^2 \sigma_Y^2$ or $\sigma_X^2 \sigma_Y^2$

1. Let $X_1, \cdots, X_n$ be a random sample of size n from $N(\mu_X, \sigma_X^2)$.

2. Let $Y_1, \cdots, Y_m$ be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

Prob. 1 Find the $100(1 - \alpha)\%$ C.I. for $\mu_X - \mu_Y$

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

1. Let $X_1, \cdots, X_n$ be a random sample of size n from $N(\mu_X, \sigma_X^2)$.

2. Let $Y_1, \cdots, Y_m$ be a random sample of size m from $N(\mu_Y, \sigma_Y^2)$.

Prob. 1 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$

When both σ_X^2 and σ_Y^2 are known

When $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, but is unknown

When $\sigma_X^2 \neq \sigma_Y^2$, both are unknown

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2, or σ_X/σ_Y

Prob. 1-1 Find the $100(1 - \alpha)\%$ C.I. for $\mu_X - \mu_Y$ with σ_X^2 and σ_Y^2 known.

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim N(0, 1)$$

$$\mathbb{P}\left(-z_{\alpha/2} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \leq z_{\alpha/2}\right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \quad , \quad (\overline{x} - \overline{y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$\square$

Prob. 1-1 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ with σ_X^2 and σ_Y^2 known.

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim N(0,1)$$

$$\mathbb{P}\left(-z_{\alpha/2} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \leq z_{\alpha/2}\right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \quad , \quad (\overline{x} - \overline{y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$\square$

Prob. 1-1 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ with σ_X^2 and σ_Y^2 known.

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim N(0,1)$$

$$\mathbb{P}\left(-z_{\alpha/2} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \leq z_{\alpha/2}\right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \quad , \quad (\overline{x} - \overline{y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$\square$

Prob. 1-1 Find the $100(1 - \alpha)\%$ C.I. for $\mu_X - \mu_Y$ with σ_X^2 and σ_Y^2 known.

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim N(0, 1)$$

$$\mathbb{P}\left(-z_{\alpha/2} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \leq z_{\alpha/2}\right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}} \quad , \quad (\overline{x} - \overline{y}) + z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$$

$\square$

Prob. 1-2 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ when $\sigma_X^2 = \sigma_Y^2 = \sigma^2$ unknown

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \sim \text{Student t-distribution } (n+m-2)$$

$$\mathbb{P}\left(-t_{\alpha/2,n+m-2} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \leq t_{\alpha/2,n+m-2}\right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n} + \frac{1}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + t_{\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n} + \frac{1}{m}} \quad , \quad (\overline{x} - \overline{y}) + t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right)$$

$\square$

Prob. 1-2 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ when $\sigma_X^2 = \sigma_Y^2 = \sigma^2$ unknown

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \sim \text{Student t-distribution } (n + m - 2)$$

$$\mathbb{P}\left(-t_{\alpha/2, n+m-2} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \leq t_{\alpha/2, n+m-2}\right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2, n+m-2} S_p\sqrt{\frac{1}{n} + \frac{1}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + t_{\alpha/2, n+m-2} S_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - t_{\alpha/2, n+m-2} S_p\sqrt{\frac{1}{n} + \frac{1}{m}} \quad , \quad (\overline{x} - \overline{y}) + t_{\alpha/2, n+m-2} S_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right)$$

$\square$

Prob. 1-2 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ when $\sigma_X^2 = \sigma_Y^2 = \sigma^2$ unknown

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \sim \text{Student t-distribution } (n + m - 2)$$

$$\mathbb{P}\left(-t_{\alpha/2,n+m-2} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \leq t_{\alpha/2,n+m-2}\right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n} + \frac{1}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + t_{\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - t_{\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n} + \frac{1}{m}} \quad , \quad (\overline{x} - \overline{y}) + t_{\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right)$$

$\square$

Prob. 1-2 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ when $\sigma_X^2 = \sigma_Y^2 = \sigma^2$ unknown

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \sim \text{Student t-distribution } (n + m - 2)$$

$$\mathbb{P}\left(-t_{\alpha/2,n+m-2} \le \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \le t_{\alpha/2,n+m-2}\right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n} + \frac{1}{m}} \le \mu_X - \mu_Y \le (\overline{X} - \overline{Y}) + t_{\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n} + \frac{1}{m}} \quad , \quad (\overline{x} - \overline{y}) + t_{\alpha/2,n+m-2}s_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right)$$

$\square$

Prob. 1-3 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ when $\sigma_X^2 \neq \sigma_Y^2$ unknown.

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \sim \text{Student t-distribution } (\nu)$$

$$\mathbb{P}\left(-t_{\alpha/2,\nu} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \leq t_{\alpha/2,\nu}\right) \approx 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left(\left(\overline{X} - \overline{Y}\right) - t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \leq \mu_X - \mu_Y \leq \left(\overline{X} - \overline{Y}\right) + t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}\right)$$

$$\left(\left(\overline{x} - \overline{y}\right) - t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}} \quad , \quad \left(\overline{x} - \overline{y}\right) + t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}\right)$$

$\square$

Prob. 1-3 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ when $\sigma_X^2 \neq \sigma_Y^2$ unknown.

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \sim \text{Student t-distribution } (\nu)$$

$$\mathbb{P}\left(-t_{\alpha/2,\nu} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \leq t_{\alpha/2,\nu}\right) \approx 1 - \alpha$$

$$\parallel$$

$$\mathbb{P}\left(\overline{X} - \overline{Y}) - t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}\right)$$

$$\left(\overline{x} - \overline{y}) - t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}} \quad, \quad (\overline{x} - \overline{y}) + t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}\right)$$

$\square$

Prob. 1-3 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ when $\sigma_X^2 \neq \sigma_Y^2$ unknown.

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \sim \text{Student t-distribution } (\nu)$$

$$\mathbb{P}\left(-t_{\alpha/2,\nu} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \leq t_{\alpha/2,\nu}\right) \approx 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}} \quad , \quad (\overline{x} - \overline{y}) + t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}\right)$$

$\square$

Prob. 1-3 Find the $100(1-\alpha)\%$ C.I. for $\mu_X - \mu_Y$ when $\sigma_X^2 \neq \sigma_Y^2$ unknown.

Sol.

$$\frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \sim \text{Student t-distribution } (\nu)$$

$$\mathbb{P}\left(-t_{\alpha/2,\nu} \leq \frac{\overline{X} - \overline{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \leq t_{\alpha/2,\nu}\right) \approx 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left((\overline{X} - \overline{Y}) - t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}} \leq \mu_X - \mu_Y \leq (\overline{X} - \overline{Y}) + t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}\right)$$

$$\left((\overline{x} - \overline{y}) - t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}} \quad , \quad (\overline{x} - \overline{y}) + t_{\alpha/2,\nu}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}\right)$$

$\square$

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2

Sol 1.

$$\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \sim \text{F-disribution } (n-1, m-1)$$

$$\mathbb{P}\left(F_{\alpha/2, n-1, m-1} \leq \frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \leq F_{1-\alpha/2, n-1, m-1} \right) = 1-\alpha$$

$$\|$$

$$\mathbb{P}\left(\frac{S_X^2}{S_Y^2} \frac{1}{F_{1-\alpha/2, n-1, m-1}} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2} \frac{1}{F_{\alpha/2, n-1, m-1}} \right)$$

$$\left(\frac{s_X^2}{s_Y^2} \frac{1}{F_{1-\alpha/2, n-1, m-1}} \quad , \quad \frac{s_X^2}{s_Y^2} \frac{1}{F_{\alpha/2, n-1, m-1}} \right)$$

$\square$

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2

Sol 1.

$$\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \sim \text{F-disribution } (n-1, m-1)$$

$$\mathbb{P}\left(F_{\alpha/2, n-1, m-1} \leq \frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \leq F_{1-\alpha/2, n-1, m-1} \right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left(\frac{S_X^2}{S_Y^2} \frac{1}{F_{1-\alpha/2, n-1, m-1}} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2} \frac{1}{F_{\alpha/2, n-1, m-1}} \right)$$

$$\left(\frac{s_X^2}{s_Y^2} \frac{1}{F_{1-\alpha/2, n-1, m-1}} \quad , \quad \frac{s_X^2}{s_Y^2} \frac{1}{F_{\alpha/2, n-1, m-1}} \right)$$

$\square$

Prob. 2 Find the $100(1-\alpha)\%$ C.I. for σ_X^2/σ_Y^2

Sol 1.

$$\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \sim \text{F-disribution } (n-1, m-1)$$

$$\mathbb{P}\left(F_{\alpha/2, n-1, m-1} \leq \frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \leq F_{1-\alpha/2, n-1, m-1} \right) = 1 - \alpha$$

$$\|$$

$$\mathbb{P}\left(\frac{S_X^2}{S_Y^2} \frac{1}{F_{1-\alpha/2, n-1, m-1}} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2} \frac{1}{F_{\alpha/2, n-1, m-1}} \right)$$

$$\left(\frac{s_X^2}{s_Y^2} \frac{1}{F_{1-\alpha/2, n-1, m-1}} \quad , \quad \frac{s_X^2}{s_Y^2} \frac{1}{F_{\alpha/2, n-1, m-1}} \right)$$

$\square$

Sol 2. Or equivalently,

$$\frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \sim \text{F-disribution } (m-1, n-1)$$

$$\mathbb{P}\left(F_{\alpha/2,m-1,n-1} \leq \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \leq F_{1-\alpha/2,m-1,n-1}\right) = 1-\alpha$$

$$\|$$

$$\mathbb{P}\left(\frac{S_X^2}{S_Y^2}F_{\alpha/2,m-1,n-1} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2}F_{1-\alpha/2,m-1,n-1}\right)$$

$$\left(\frac{S_X^2}{S_Y^2}F_{\alpha/2,m-1,n-1} \quad , \quad \frac{S_X^2}{S_Y^2}F_{1-\alpha/2,m-1,n-1}\right)$$

$\square$

Recall: $\quad F_{\alpha,m,n} = \frac{1}{F_{1-\alpha,n,m}}$

Sol 2. Or equivalently,

$$\frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \sim \text{F-disribution } (m-1, n-1)$$

$$\mathbb{P}\left(F_{\alpha/2,m-1,n-1} \leq \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \leq F_{1-\alpha/2,m-1,n-1} \right) = 1-\alpha$$

$$\|$$

$$\mathbb{P}\left(\frac{S_X^2}{S_Y^2} F_{\alpha/2,m-1,n-1} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2} F_{1-\alpha/2,m-1,n-1} \right)$$

$$\left(\frac{s_X^2}{s_Y^2} F_{\alpha/2,m-1,n-1} \quad , \quad \frac{s_X^2}{s_Y^2} F_{1-\alpha/2,m-1,n-1} \right)$$

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Recall: $\quad F_{\alpha,m,n} = \dfrac{1}{F_{1-\alpha,n,m}}$

Examples from the book...