Probability Exercises

Look at the exercises below, several from here or from 50cpip.
Try to solve as many as possible before our meeting.

1. Imagine you have an urn filled with 100 balls, some red and some green. You know that someone determined the number of red balls by picking a number between 0 and 100 from a hat. You reach into the urn and pull out a ball, it's red. If you now pull out a second ball, is it more likely to be red or green? What is the probability it is red?

2. [50cpip #22] In an election, a ballot box contains a votes for candidate A, and b votes for candidate B, where $a > b$. If we draw ballots from the box in random order to count them, what is the probability that we see a tie at least once, after the first ballot is drawn? (Find a formula involving a and b. For example, the probability is 8/10 when $a = 3$ and $b = 2$.)

3. [50cpip #35] From where he stands, one step toward the cliff would send the drunken man over the edge. He takes random steps, either toward or away from the cliff. At any step his probability of taking a step away is 2/3, of a step toward the cliff is 1/3. What is his chance of escaping the cliff?

4. Let C be the unit circle $x^2 + y^2 = 1$. A point p is chosen randomly on the circumference of C, and another point q is chosen randomly from the interior of C (both are chosen independently and uniformly). Let R be the rectangle with sides parallel to the x and y-axes with diagonal pq. What is the probability that no point of R lies outside of C?

5. Suppose $n \geq 3$ points are drawn at random from the circumference of a circle. They define a convex n-gon. What is the probability that the n-gon contains the center of the circle?

6. [1992 A-6] Four points are chosen at random on the surface of a sphere, uniformly and independently. What is the probability that the center of the sphere lies in the tetrahedron defined by the four points?

7. [2005 A-6] Suppose $n \geq 4$ points are drawn at random from the circumference of a circle. They define a convex n-gon. What is the probability that the convex n-gon has at least one acute vertex angle?